Math 082
Solutions to
Fall 2019 Test 2
(practice exam for
Winter 2020 Test 2)

1. Plug $(-1, 3)$ into the equation.

$2(-1) + 3(3) = 7$

$-2 + 9 = 7$

$7 = 7$ ✓

$(-1, 3)$ is a solution to the equation

2. Use the slope equation for two points

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$, $\underset{x_1 \quad y_1}{(-2, 6)}$ & $\underset{x_2 \quad y_2}{(3, -2)}$

$m = \dfrac{-2-6}{3-(-2)} = \dfrac{-8}{3+2}$

$m = \dfrac{-8}{5}$

3. Plug $(2, -4)$ into the first equation

$\frac{1}{3}(2) - (-4) = -1$

$\frac{2}{3} + 4 = -1$ Get a common denominator

$\frac{2}{3} + \frac{12}{3} = -1$

$\frac{14}{3} \neq -1$

Therefore, $(2, -4)$ is not a solution to the system of equations, and we don't have to test the second equation.

4. $(-5z^5)(3z^3)(2z^{-2}) = (-5 \cdot 3 \cdot 2)(z^5 z^3 z^{-2})$ Add Exponents

$= -30z^6$

5. $(-6)^{-2} = \frac{1}{(-6)^2} = \frac{1}{(-6)(-6)} = \frac{1}{36}$

6. Find the slope between the two points $\overset{x_1\ y_1}{(4,5)}$ + $\overset{x_2\ y_2}{(2,-3)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{2-4} = \frac{-8}{-2} = 4$

Plug slope and one point into point-slope equation

$y - y_1 = m(x - x_1)$

$y - 5 = 4(x - 4)$ Simplify

$y - 5 = 4x - 16$

$+5 \qquad +5$

$y = 4x - 11$

7. Find the slope of the equation by putting it into slope-intercept form $(y = mx + b)$ where m is the slope

$5x - 3y = -4$

$-5x \qquad -5x$

$\frac{-3y}{-3} = \frac{-5x}{-3} - \frac{4}{-3}$

$y = \frac{5}{3}x + \frac{4}{3}$

$m = \frac{5}{3}$

The perpendicular slope is the negative-reciprocal of the slope we just found.

$m_{\perp} = -\frac{3}{5}$

7 cont. Plug the new slope and point into point-slope form.

$y - y_1 = m_{\perp}(x - x_1)$ where $m_{\perp} = -\frac{3}{5}$ & $(\overset{x_1}{-5}, \overset{y_1}{2})$

$y - 2 = -\frac{3}{5}(x - (-5))$

$y - 2 = -\frac{3}{5}x - 3$

$+2 \qquad\qquad +2$

$y = -\frac{3}{5}x - 1$

8. Find the x-intersept by setting $y = 0$

$x - 2(0) = -4$

$x = -4 \longrightarrow (-4, 0)$

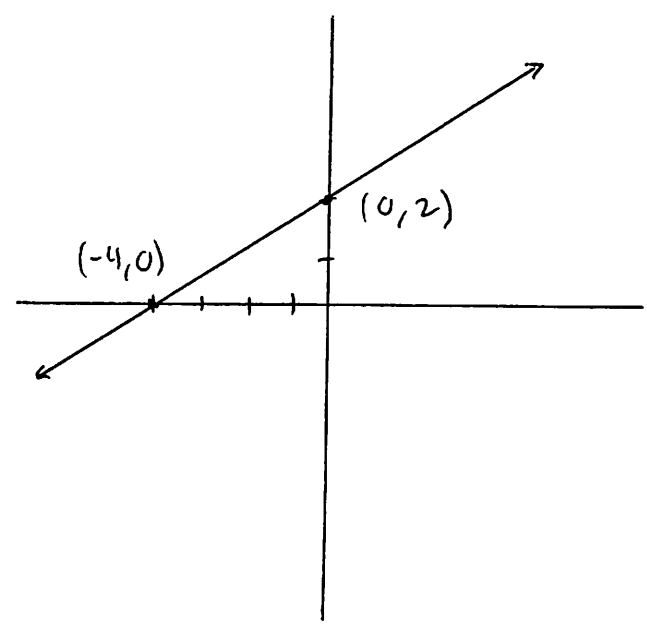
Find the y-intersept by setting $x = 0$

$0 - 2y = -4$

$\frac{-2y}{-2} = \frac{-4}{-2}$

$y = 2 \longrightarrow (0, 2)$

Now Graph both points



9. $\dfrac{(2a^7 b^{-8})^5}{2^3 a^{-9} b^{-3}} = \dfrac{2^5 a^{35} b^{-40}}{2^3 a^{-9} b^{-3}}$ Distribute and multiply top exponent

$= 2^{5-3} a^{35-(-9)} b^{-40-(-3)}$ Subtract exponents

$= 2^2 a^{44} b^{-37}$ Simplify

$= \dfrac{4a^{44}}{b^{37}}$

10. $(9m^2 - 8m - 13) - (m^2 - 8m - 24)$ Distribute the negative

$9m^2 - \cancel{8m} - 13 - m^2 + \cancel{8m} + 24$ Simplify

$8m^2 + 11$

11. $-\frac{1}{3}x^2(3x^2 - 12x + 9)$ Distribute

$-x^4 + 4x^3 - 3x^2$

12. $(5m + 4)(m^2 - 2m + 1)$ Distribute first and second term through

$5m^3 - 10m^2 + 5m + 4m^2 - 8m + 4$ simplify

$5m^3 - 6m^2 - 3m + 4$

13. $(2y + w)^2$ Use the formula $(a+b)^2 = a^2 + 2ab + b^2$

$(\overset{a}{2y} + \overset{b}{w})^2 = (2y)^2 + 2(2y)(w) + (w)^2$ simplify

$= 4y^2 + 4yw + w^2$

OR $(2y + w)^2 = (2y + w)(2y + w)$ FOIL

$= 4y^2 + 2yw + 2yw + w^2$ simplify

$= 4y^2 + 4yw + w^2$

14.) $(5-2x)(5+2x)$. Two ways of approaching

A.) FOIL:

$$\begin{array}{cccc} F & O & I & L \\ 5\cdot 5 & +\,5(2x) & +\,(-2x)(5) & +\,(-2x)(2x) \end{array}$$

$$= 25 + 10x - 10x - 4x^2$$

$$= 25 - 4x^2.$$

B.) Recognize difference of two squares pattern: $(a+b)(a-b) = a^2 - b^2$.

$$\Longrightarrow \quad (5-2x)(5+2x) = 5^2 - (2x)^2 = \boxed{25 - 4x^2}$$

15.) $$\frac{32x^4 - 12x^3}{4x^3} = \frac{1}{4x^3}(32x^4 - 12x^3),$$ since division is the same as multiplication by the reciprocal.

$$= \frac{1}{4x^3}(32x^4 - 12x^3)$$ Distribute.

$$= \frac{32x^4}{4x^3} - \frac{12x^3}{4x^3} = 8x^{4-3} - 3x^{3-3} = 8x^1 - 3x^0 = \boxed{8x - 3}$$

16.) $(8 \times 10^7)(1.5 \times 10^{-3})$

$= (8 \cdot 1.5)(10^7 \cdot 10^{-3})$ Rewrite via associative property.

$= 12 \times 10^{7-3}$

Multiply.

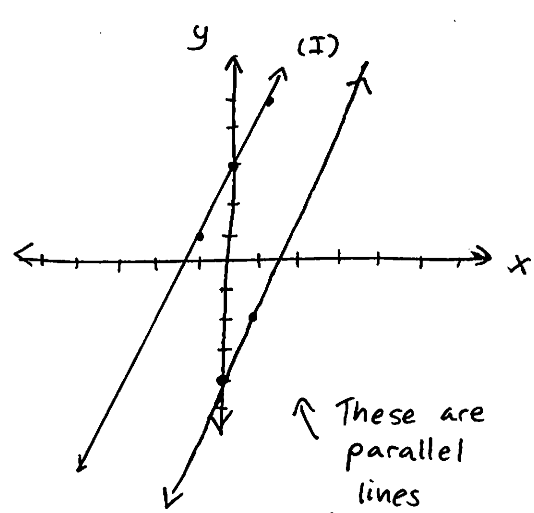
$= 12 \times 10^4$

$= \boxed{1.2 \times 10^5}$ Recall the first term must be between 1 and 10.

How? $12 = 1.2 \cdot 10$

$\Longrightarrow \quad 12 \cdot 10^4 = 1.2 \cdot 10^1 \cdot 10^4 = 1.2 \times 10^5.$

For Problem 17.



17.) $$\begin{cases} -2x + y = 3 & \text{(I)} \\ 4x - 2y = 8 & \text{(II)} \end{cases}$$ Write each in slope intercept form OR plot by intercepts.

(I) $-2x + y = 3$

$y = 2x + 3$

(II) $4x - 2y = 8$

$-2y = -4x + 8 \Rightarrow y = 2x - 4.$

These are parallel lines (same slope) $\Rightarrow$ No intersections $\Rightarrow$ NO SOLUTION

18.) $\begin{cases} 6x + 3y = -12 & (\text{I}). \\ -3x + y = 1 & (\text{II}) \end{cases}$ ← The y here has a coefficient of 1 → good candidate for substitution.

(II): $-3x + y = 1.$

$y = 3x + 1.$ → Now substitute this into (I).

(I): $6x + 3y = -12$

$6x + 3(3x+1) = -12$ (Substitution).

$6x + 9x + 3 = -12$

$15x + 3 = -12$

$15x = -15$

$x = -1.$ → with x, subsitute value into (I) or (II).

(II): $y = 3x + 1$

$y = 3(-1) + 1$ ⇒ $\boxed{(-1, -2)}$ ∎

$y = -3 + 1 = -2.$

19.) $\begin{cases} x + 2y = 2 & (\text{I}) \\ 3x - y = -22. & (\text{II}). \end{cases}$ Multiply (II) by 2 to get a coeff. of -2 on the y to cancel w/ (I).

$$\begin{array}{lrcl} (\text{I}) & x + 2y & = & 2 \\ +2\,(\text{II}) & 6x - 2y & = & -44 \\ \hline & 7x + 0 & = & -42 \\ & x & = & -6. \end{array}$$

→ Substitute back into (I) or (II).

(II) $3x - y = -22$

$3(-6) - y = -22$

$-18 - y = -22$

$-y = -4$

$y = 4$

Solution: $\boxed{(-6, 4)}$ ∎

20.) "Sum of two numbers is 151" ⇒ $x + y = 151$ (I) our first equation.

Sum ↑ add; two numbers: Call them x, y. is 151: $= 151$

"Difference of the two #'s is 25" ⇒ $x - y = 25$ (II). Second equation.

Difference ↓ Subtract. two #'s: still x, y; is ↓ $=$; 25 ↓ 25

CONTINUED.

20. CONTINUED) Solve $\begin{cases} x+y = 151 & (\text{I}) \\ x-y = 25 & (\text{II}). \end{cases}$ Use elimination.

(I)+(II)

$\Rightarrow \quad (x+x)+(y-y) = 151+25$

$$2x = 176.$$

$$x = 88. \longrightarrow \text{Substitute into (I).}$$

$x+y = 88+y = 151$

$y = 63.$

$\Rightarrow$ Solution: The two numbers are $\boxed{88 \text{ and } 63.}$

EXTRA CREDIT

EC.1.) Call c = # cardholders' tickets, n = # non-cardholders' tickets.

"287 tickets sold" $\Rightarrow \quad c+n = 287.$ (I).

"total money collected was \$149.50" $\Rightarrow \quad \underbrace{0.50c}_{\text{money from cardholders}} + \underbrace{0.55n}_{\text{money from non-cardholders}} = \underbrace{149.50}_{\text{total.}}$ (II).

Solve $\begin{cases} c+n = 287 & (\text{I}) \\ 0.5c + 0.55n = 149.5 & (\text{II}). \end{cases}$ $\longrightarrow$ Substitution via (I) $287-n = c.$

$\Rightarrow$ (II) $0.5c + 0.55n = 149.5$

$$0.5(287-n) + 0.55n = 149.5$$

$$143.5 - 0.5n + 0.55n = 149.5$$

$$0.05n = 6$$ Simplify and put all constants on the right.

$$n = 120.$$

$\Rightarrow$ Put into eq. (I): $c = 287-120 = 167$

$\boxed{\text{\# cardholders' tickets : 167, \# non-cardholders' tickets = 120}}$

EC.2) Recall problem 14: This is a difference of two squares form:

$(6m^2+10)(6m^2-10) = (6m^2)^2 - 10^2$

$= \boxed{36m^4 - 100}$