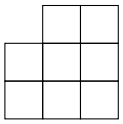
Saginaw Valley State University

2008 Math Olympics – Level I

1. How many squares are there altogether in this diagram?

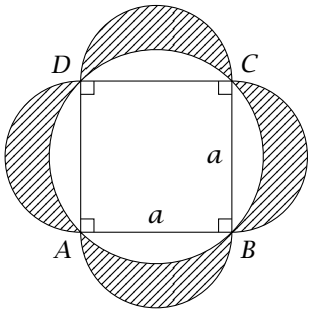


(a) 8 **(b)** 9 **(c)** 10 **(d)** 11 **(e)** 12

2. How many different ways are there to pay a \$9.75 bill if only dimes and quarters are available?

(a) 39 **(b)** 19 **(c)** 20 **(d)** 40 **(e)** None of the above

3. The square $ABCD$, with a side a, is inscribed in a circle. Outside of each side of the square, there are semicircles as shown. What is the area of the shaded region?



(a) $\dfrac{\pi a^2}{2}$

(b) a^2 **(c)** $\pi a^2\left(1-\dfrac{\sqrt{2}}{2}\right)$

(d) $a^2\left(1-\dfrac{\sqrt{2}}{2}\right)$ **(e)** None of the above

4. The owners of an apartment complex find that if they charge \$500 per month for rent they can rent out all 200 units, but for every \$10 increase in rent, they lose one renter. How much should they charge per month to maximize the monthly income?

(a) \$125 **(b)** \$1250 **(c)** \$2500 **(d)** \$500 **(e)** None of the above

5. In a small school 7 students are in the Math class, 7 are in the Science class, and 8 are in the Music class. Three students are in both Math and Science, five are in both Science and Music, and four are in both Math and Music. Two students are taking all three classes. How many students are there taking at least one of the three classes?

(a) 10 **(b)** 12 **(c)** 22 **(d)** 36 **(e)** None of the above

6. If the parabola $y = x^2 + bx + c$ has an x-intercept at 2 and a y-intercept at -8, what is its vertex?

(a) $(-1,-9)$ **(b)** $(-1,-5)$ **(c)** $(1,-5)$ **(d)** $(1,9)$ **(e)** None of the above

7. The product of $\sqrt{5}$ and $\sqrt[3]{7}$ is

(a) $\sqrt[5]{35}$ **(b)** $\sqrt[6]{35}$ **(c)** $\sqrt[5]{6125}$ **(d)** $\sqrt[6]{6125}$ **(e)** None of the above

8. A circular table has exactly 60 chairs around it. There are N people seated around the table. The next person coming to the table will have to be seated next to an occupied seat. Find the smallest possible value of N.

   (a) 15 **(b)** 20 **(c)** 30 **(d)** 40 **(e)** 58

9. Simplify the following expression:

$$\frac{\frac{yz}{x} - \frac{xz}{y}}{\frac{x-2y}{yz} + \frac{y}{xz}}$$

   (a) $\dfrac{(y-x)(y+x)}{y(x^2-2x+y)}$ **(b)** $-\dfrac{z^2}{2x}$ **(c)** 0 **(d)** $-\dfrac{z^2(x+y)}{(x-y)}$

   (e) None of the above

10. For how many integers n between 1 and 100 can $x^2 + x - n$ be factored into the product of two linear factors with integer coefficients?

    (a) 0 **(b)** 1 **(c)** 2 **(d)** 9 **(e)** 10

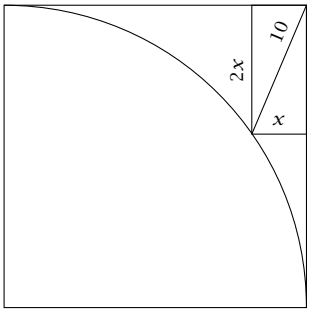
11. The strange operation $*$ is defined to be:

$$a * b := \frac{a}{a + \frac{1}{b}}$$

    where a and b are real numbers. Find $5 * (1 * 3)$.

    (a) $\frac{15}{19}$ **(b)** $\frac{5}{7}$ **(c)** $\frac{15}{16}$ **(d)** $\frac{20}{23}$ **(e)** None of the above

12. A quarter circle is inscribed in a square. A rectangle with dimensions x and $2x$ (as shown) is inscribed in the region of the square outside of the circle, touching the circle, as shown. What is the radius of the circle if the diagonal of the rectangle equals 10 units?



    (a) $10\sqrt{5}$

    (b) $2\sqrt{5}$ **(c)** 20 **(d)** $6\sqrt{5}$ **(e)** None of the above

13. The circumference of Gus's pizza is 30% greater than the circumference of Max's pizza. The area of Gus's pizza is what % greater than the area of Max's pizza?

    (a) 30% **(b)** 60% **(c)** 69% **(d)** 90% **(e)** None of the above

14. If $xy = 10$ and $x^2y + xy^2 + x + y = 99$, find $x^2 + y^2$.

    (a) 61 **(b)** 71

    (c) 81 **(d)** Not enough information given

    (e) None of the above

15. The digits of the whole numbers from 1 to 99 are concatenated in order to form thye number N:

    $$N = 1234567891011121314\ldots979899$$

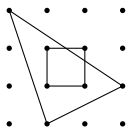
    Which of the following is true?

    (a) N is divisible by 3 but not by 6 and 9 **(b)** N is divisible by 3 and 6 but not by 9

    (c) N is divisible by 3 and 9 but not by 6 **(d)** N is not divisible by any of 3, 6 or 9

    (e) None of the above

16. The dots are one unit apart. What is the area (in square units) of the region that is common to both the square and the triangle?



    (a) $\frac{9}{10}$ **(b)** $\frac{15}{16}$ **(c)** $\frac{8}{9}$ **(d)** $\frac{11}{12}$ **(e)** $\frac{14}{15}$

17. A used car salesman sells 2 cars for \$6000 each. He makes a 25% profit on the first car and takes a 20% loss on the second car. Which of the following is true?

    (a) The salesman has a net gain of 5% on the two transaction combined.

    (b) The salesman has a net gain of \$300 on the two transaction combined.

    (c) The salesman has a net loss of \$300 on the two transaction combined.

    (d) There is not enough information given to determine his net gain or loss.

    (e) None of the above

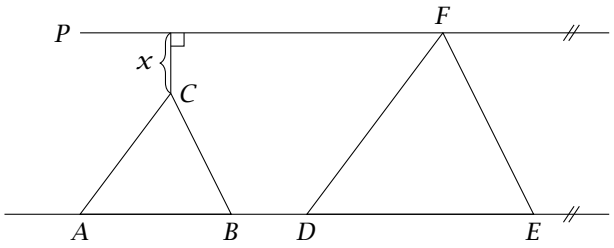
18. Joe wants to cook for his friends using a recipe from a cookbook. He needs to feed a lot of people, so he triples the recipe. At the dinner only two thirds of his dish is eaten. Then as he is cleaning up he drops one fourth of the left overs on the floor, but he keeps the rest. If the original cookbook recipe makes 8 servings, how many servings will the left overs make?

    (a) 2 **(b)** 4 **(c)** 6 **(d)** 8 **(e)** None of the above

19. Max and Gus start their new jobs on the same day. In Max's schedule, one rest-day follows every three work-days. In Gus's schedule, three rest-days follow every seven work-days. How many times during their first 1000 days do they both have a rest-day on the same day?

    (a) 48 **(b)** 50 **(c)** 72 **(d)** 75 **(e)** 100

20. Two similar triangles, $\triangle ABC$ and $\triangle DEF$ have bases on line AE. The area of $\triangle ABC$ is 100 square units, while the area of $\triangle DEF$ is 144 square units. The side DE is 6 units long. The lines AE and PF are parallel. Find x.



(a) $14.\overline{6}$ **(b)** 28

(c) 19.2 **(d)** 8 **(e)** None of the above

21. How many ordered pairs (m, n) of positive integers are solutions to

$$\frac{4}{m} + \frac{2}{n} = 1?$$

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** None of the above

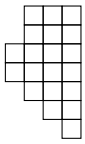
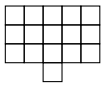
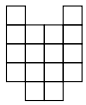
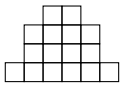
22. A bag contains a mix of red and blue marbles. If one red marble is removed, then one-seventh of the remaining marbles are red. If two blue marbles are removed instead of one red, then one-fifth of the remaining marbles are red. How many marbles were in the bag originally?

(a) 8 **(b)** 22

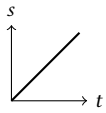
(c) 36 **(d)** 57

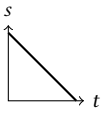
(e) Not enough information given

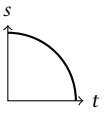
23. Which shape *cannot* be filled, without any overlapping, using copies of the tile showed on the right?

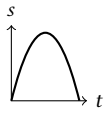
(a)  **(b)**  **(c)**  **(d)**  **(e)** 

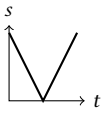
24. I throw a ball vertically up into the air. Which of these graphs might reasonably show the speed s of the ball as a function of time t since leaving my hand?

(a) 

(b) 

(c) 

(d) 

(e) 

25. If you travel 60 miles at 20 miles per hour and then travel 60 miles at 30 miles per hour, what is your average speed for the entire trip?

(a) 60 mph **(b)** 25 mph **(c)** 23 mph **(d)** 50 mph **(e)** None of the above