

# 2000 MATH OLYMPICS

## LEVEL I

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1. Find the equation of a line through the points  $(-1,4)$  and  $(6,2)$ .

a.  $2x - 26 = -7y$

b.  $7y = 2x + 2$

c.  $2x - 7y = 26$

d.  $4x = -y$

e. None of the above

2. A farmer wishes to erect a fence-line 640 feet long. If he wants a post every 16 feet, how many posts does he need?

a. 39

b. 40

c. 41

d. 42

e. None of the above

3. The area of the larger square in the figure below is  $A$ . Find the area of the smaller square.

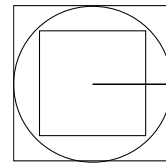
a.  $\frac{A}{2}$

b.  $\frac{A}{\sqrt{2}}$

c.  $\sqrt{\frac{A}{2}}$

d.  $\frac{A}{\sqrt{3}}$

e. None of the above



4. A man purchased a radio for \$55, gave the clerk a \$100 bill and received change. Later the man returned and used a \$20 bill to buy \$12 worth of speaker wire for the radio. Hours later, the storekeeper was informed by the bank that the \$100 and the \$20 bill were counterfeit.

The wholesale cost of the radio was \$33. The wiring had a \$7 wholesale cost. Ignoring the lost profit and the store overhead, how much is the storekeeper out, in dollars?

a. \$187

b. \$120

c. \$107

d. \$93

e. None of the above

5. If a square  $DEFG$  is drawn on the outside of regular pentagon  $ABCDE$ , how big is  $\angle EAF$ ?

a.  $9^\circ$

b.  $12^\circ$

c.  $15^\circ$

d.  $18^\circ$

e. None of the above

6. Solve  $3(2r + 1) + 1 = 27 - 5(r - 2)$ .

- a.  $3/7$       b.  $23/11$       c.  $-8/13$       d.  $12/13$       e. None of the above

7. Simplify  $\frac{y^2z}{2y^3} \div \frac{9z^5}{8y^4}$ .

- a.  $\frac{4y^3}{9z^4}$       b.  $\frac{9z^6}{16}$       c.  $\frac{9z^4}{4y^8}$       d.  $\frac{2y^8}{3z^4}$       e. None of the above

8. The fourth number in the list is missing. Find the sum of the digits of this missing number.

5      11      23      ???      95      191

- a. 5      b. 11      c. 12      d. 14      e. None of the above

9. If  $f(x) = x^2 + 3$ , then  $f(x + 2)$  is equal to:

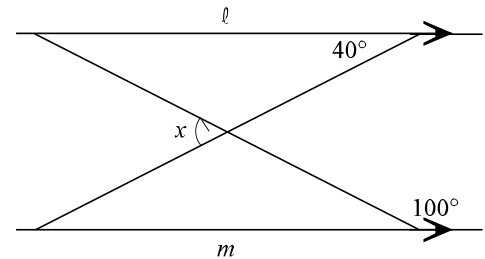
- a.  $x^2 + 4x + 3$       b.  $x^2 + 4x + 7$       c.  $x^2 + 7$   
 d.  $x^2 + 2x + 3$       e. None of the above

10. Four of the following five points, with the coordinates given, lie on a single straight line. Which one does not lie on the line?

- a. (6,11)      b. (5,5)      c. (4,7)      d. (8,15)      e. (2,3)

11. If line  $\ell$  is parallel to line  $m$ , how big is the angle marked  $x$ ?

- a.  $40^\circ$       b.  $80^\circ$       c.  $100^\circ$   
 d.  $120^\circ$       e. None of the above



12. In a triangle PQR, no angle is larger than  $85^\circ$ . What is the smallest possible size of the smallest angle?
- a.  $0^\circ$       b.  $1^\circ$       c.  $5^\circ$       d.  $10^\circ$       e. None of the above
13. The range, expressed in interval notation, of the function  $g(x) = 3 + \sqrt{x-4}$  is:
- a.  $[0, \infty)$       b.  $[3, \infty)$       c.  $[4, \infty)$       d.  $(-\infty, \infty)$       e. None of the above
14. In a bag there are six white balls and four black balls. Two balls are taken out at random, one at a time, without replacement. What is the probability that the *second* ball taken out is black?
- a.  $2/5$       b.  $4/9$       c.  $2/3$       d.  $1/3$       e. None of the above
15. How many of the following statements are true?
- i)  $50\%$  of  $80 = 80\%$  of  $50$   
ii)  $3/4$  of  $72 = 54$   
iii)  $2/91 > 2/93$   
iv)  $3.106 \div 1.6 > 2$
- a. none      b. one      c. two      d. three      e. four
16. Two adjacent sides of a parallelogram are 12 m and 10 m in length. If the included angle is 60 degrees, what is the area of the parallelogram?
- a. 120 square meters      b.  $60\sqrt{3}$  square meters      c.  $60\sqrt{2}$  square meters  
d. 60 square meters      e. None of the above
17. What is the smallest prime number that is a factor of the sum of  $7^{35}$  and  $3^{115}$ ?
- a. 2      b. 3      c. 5      d. 7      e. None of the above
18. A one foot metal rod is anchored at both ends. On a hot day it expands one inch and buckles in the middle. How high off the ground is it at its midpoint?

- a.  $\frac{1}{2}$  inch      b. 1 inch      c. 2 inches      d.  $2\frac{1}{2}$  inches      e. None of the above

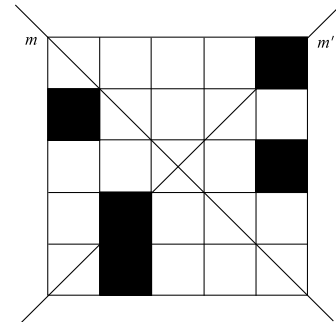
19. Caleb walks at 4 km per hour and runs at 6 km per hour. If he saves 3 minutes and 45 seconds by running instead of walking to school in the mornings, how far does Caleb live from school?

- a. 0.375 km      b. 0.75 km      c. 1.0 km      d. 1.5 km      e. None of the above

20. Sam-the-Super-Snail is climbing a vertical stone 1 meter high. He climbs at a steady speed of 30 cm per hour, but each time the church clock strikes the shock causes him to slip down 1 cm. The clock strikes the hour, so at 1 o'clock he would slip back 1 cm, at 2 o'clock he would slip back 2 cm and so on. If he starts to climb just after the clock strikes 3:00 p.m., when will he reach the top?

- a. 3:50 p.m.      b. 6:20 p.m.      c. 6:50 p.m.      d. 7:20 p.m.      e. None of the above

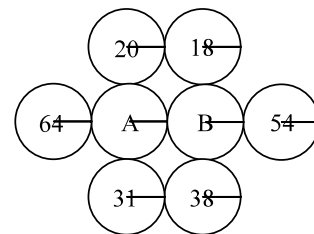
21. If the shading of squares is continued so that line  $m$  and line  $m'$  become the lines of symmetry of the completed diagram, what is the largest possible number of squares left unshaded?



- a. 5                      b. 7                      c. 9  
d. 11                    e. None of the above

22. Ali (A) and Baba (B) are surrounded by six thieves (as shown). The thieves' ages are given. Ali's age is the average of his four nearest neighbors, and so is Baba's. How old is Baba?

- a. 37                    b.  $37\frac{1}{2}$                     c. 38  
d.  $38\frac{1}{3}$                     e. None of the above



23. Which interval(s) gives all the values of  $a$  such that  $2x^2 + ax + 8 = 0$  has no real solutions?
- a.  $(-\infty, -8), (8, \infty)$    b.  $(-\infty, 8)$    c.  $(0, 8)$    d.  $(-8, 8)$    e. None of the above
24. A car can go  $r$  miles on  $s$  gallons of gasoline. How many gallons of gas would it need for a  $t$ -mile trip?
- a.  $\frac{st}{r}$    b.  $\frac{rs}{t}$    c.  $\frac{rt}{s}$    d.  $\frac{t}{rs}$    e. None of the above
25. If 64 men and 64 women each received 64 pennies, what was the total number of pennies received?
- a.  $2^6$    b.  $2^{12}$    c.  $2^{13}$    d.  $2^{24}$    e. None of the above