

- 1) (A) The LCM of 6, 9, and 10 is 90, so lockers 90, 180, 270, 360, and 450 will have all three decals.
- 2) (E) Multiplying numerator and denominator by a^2b^2 gives $\frac{a^2b^2(a-b)}{b^2-a^2} = \frac{a^2b^2(a-b)}{(b-a)(b+a)} = -\frac{a^2b^2}{a+b}$.
- 3) (B) $x = 1$ is clearly a solution and long or synthetic division or trial-and-error gives the factorization $(x-1)(2x^2+7x+5) = 0$, or $(x-1)(2x+5)(x+1) = 0$. The sum of the solutions is $1 + (-2.5) + (-1) = -2.5$.
- 4) (D) There are $5 \cdot 4 = 20$ possible orderings of 2 people from the 5, and two of these (W1 W2, or W2 W1, where W1 and W2 are the two women) consist of the two women only. Two of the 20 gives 10%.
- 5) (E) Using a property of the log function, this simplifies to $\log\left(\frac{x-6}{x+3}\right) = 1$, and so $\frac{x-6}{x+3} = 10$ since it is base 10. Therefore, $x-6 = 10x+30$ or $x = -4$. Since -4 is not in the domain, there is no solution.
- 6) (C) Using G, B, M, and P for the number of glasses, bowls, mugs, and plates, respectively, we have the three equations $G+B=M$, $B=G+P$, $3P=2M$. From Eq 1, $2G+2B=2M$, and combining with Eq. 3, then, $3P=2G+2B$. Finally, from Eq. 2, $P=B-G$, so $3(B-G)=2G+2B$, or $B=5G$.
- 7) (C) Let ℓ be the length of the level road in miles and s the length of the trail. Since they travel each twice, the total distance is $2\ell+2s$. Using time is distance divided by rate, we have $\frac{\ell}{4} + \frac{s}{3} + \frac{s}{6} + \frac{\ell}{4} = 6$, or $\frac{\ell}{2} + \frac{s}{2} = 6$. Multiplying both sides by 4, $2\ell+2s=24$.
- 8) (D) $\log_2 \frac{1}{y} = \log_2 1 - \log_2 y = -\log_2 y$, and by taking the log of both sides of $2^x = y$, we get $x = \log_2 y$. So, $\log_2 \frac{1}{y} = -\log_2 y = -x$.
- 9) (D) A graph of $y = \cos \theta$, shows this characteristic.
- 10) (A) Since it is absolute value, $3x+1 > x+1$ or $3x+1 < -(x+1)$, and solving each inequality, we get $2x > 0$ and $4x < -2$, respectively. So, $x > 0$ or $x < -\frac{1}{2}$.
- 11) (A) Since the sine and cosine functions both have ranges between -1 and 1, inclusive, the product of these functions must also be between -1 and 1, inclusive. So, the only possibility is $\sin A \cos B = 1$, which can only happen when $\sin A = \cos B = 1$ or $\sin A = \cos B = -1$.
- 12) (D) Since the part remaining is similar to the whole pyramid and the builder had $25/100 = 1/4$ of the height to complete, he/she had $(1/4)^3 = 1/64$ of the volume to complete. Since the builder failed to complete $1/64^{\text{th}}$ of the pyramid, he/she completed $63/64^{\text{th}}$ of the job.
- 13) (B) The minute hand is on the 9 and the hour hand is $45/60 = 3/4$ of the way past the seven, or $1/4$ th of the way to the 8. Since there are $360/12 = 30$ degrees between two consecutive numbers on the clock, we have 30 (between the 8 and the minute hand) plus $\frac{1}{4} \cdot 30 = 7.5$ (between the 8 and the hour hand) degrees.

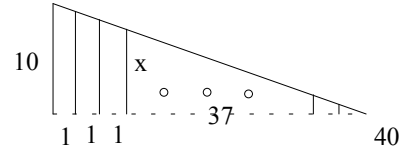
- 14) (C) There are $10 \cdot 9 \cdot 8 \cdot 7$ possible committees (orderings) of 4 people from the 10. For any of 6 orderings that have two men and two women (MMWW, MWWM, MWMW, WMMW, WWMM, WMWM), there are $5 \cdot 4 \cdot 5 \cdot 4$ possibilities for choosing the two men and two women. So, the probability is $\frac{6 \cdot 5 \cdot 4 \cdot 5 \cdot 4}{10 \cdot 9 \cdot 8 \cdot 7} = \frac{3 \cdot 4 \cdot 5 \cdot 4}{9 \cdot 8 \cdot 7} = \frac{5 \cdot 2}{3 \cdot 7} = \frac{10}{21}$.

- 15) (C) Since the streamers (vertical lines) are parallel to the pole (10 ft), we can use repeated similar triangles to get $\frac{10}{40} = \frac{x_1}{39}$,

$$\frac{10}{40} = \frac{x_2}{38}, \frac{10}{40} = \frac{x_3}{37} \quad (x_3 = x \text{ in the picture}), \frac{10}{40} = \frac{x_4}{36}, \dots, \frac{10}{40} = \frac{x_{37}}{3},$$

$$\frac{10}{40} = \frac{x_{38}}{2}, \frac{10}{40} = \frac{x_{39}}{1}. \text{ So,}$$

$$x_1 + x_2 + \dots + x_{39} = \frac{10}{40} (39 + 38 + 37 + \dots + 2 + 1) = \frac{1}{4} \cdot \frac{40 \cdot 39}{2} = 195.$$



- 16) (A) From the figure, $x + 4y + 11x = 12x + 4y = 180$. From $x + y = 20$, $y = 20 - x$, so $12x + 4(20 - x) = 180$. Solving, we get $8x = 100$, or $x = 12.5$.

- 17) (D) Getting a common denominator, we have

$$\frac{(1 + \sin t)(1 + \sin t) + \cos t \cdot \cos t}{\cos t(1 + \sin t)} = \frac{1 + 2 \sin t + \sin^2 t + \cos^2 t}{\cos t(1 + \sin t)} = \frac{2 + 2 \sin t}{\cos t(1 + \sin t)} = \frac{2}{\cos t} = 2 \sec t.$$

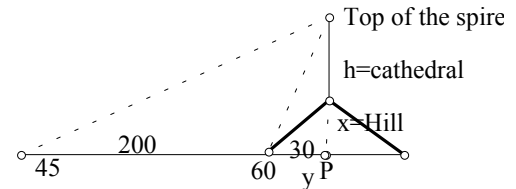
- 18) (B) From the figure and the Pythagorean Theorem, $(r - 8)^2 + (r - 9)^2 = r^2$. Expanding, we get, $2r^2 - 34r + 145 = r^2$, or $r^2 - 34r + 145 = (r - 29)(r - 5) = 0$. From the placement of P in the figure, only $r = 29$ will work.

- 19) (D) If we let h be the height of the cathedral to the top of the spire, x the height of the hill, and y the distance from the base of the hill to the point P, then

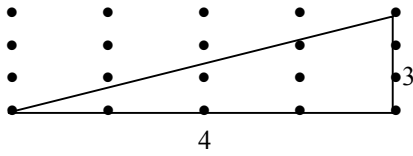
$$\tan 30 = \frac{x}{y}, \quad \tan 60 = \frac{h + x}{y}, \quad \tan 45 = \frac{h + x}{200 + y}. \text{ From these equations,}$$

$y = \sqrt{3} x$, $\sqrt{3} y = h + x$, $200 + y = h + x$, respectively. The first 2 equations combine to give $3x = x + h$, or $x = h/2$, and substituting into the third equation, $y = 3h/2 - 200$. Finally, substituting these expressions of x and y in

terms of h into equation 2, So, $\frac{3\sqrt{3} h}{2} - 200\sqrt{3} = \frac{3h}{2}$, or $h = \frac{2}{3\sqrt{3} - 3} \cdot 200\sqrt{3} = \frac{400\sqrt{3}(3\sqrt{3} + 3)}{18} = 200 + \frac{200\sqrt{3}}{6}$



- 20) (B) There are ${}_{20}C_2 = 190$ different segments connecting 2 of the 20 points. The segments of integer length are from the four horizontal sequences of five points, $4 \cdot {}_5C_2 = 40$, and the 5 vertical sequences of four points, $5 \cdot {}_4C_2 = 30$. The remaining lengths come from diagonal segments, and from the Pythagorean theorem, we need a hypotenuse of integer (rational) length. These can only come from the 3-4-5 triangles, where the diagonal is length 5, and there are 2 of these. So the probability is $72/190 = 36/95$.

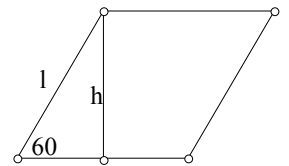


- 21) (D) Applying the log function to both sides, $(x + 1) \log 2 = (2x - 3) \log 3$. Expanding,

$$2x \log 3 - x \log 2 = \log 2 + 3 \log 3, \text{ so } x = \frac{\log 2 + 3 \log 3}{2 \log 3 - \log 2}.$$

- 22) (D) Every cubic polynomial has 3 roots, call them a, b, and c (counting complex roots), and since the graph of the second equation is just the graph of the first shifted to the right 3 units, the roots of the second are a+3, b+3, and c+3. So the sum of the roots is a+b+c+9 = (-4) + 9 = 5.

- 23) (D) The volume is the area of the base times the height of the prism. The area of the base is the area of the rhombus (shown), which is base times height = $1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$. The height of the prism, from the figure, is $\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3}{4}$. Therefore, the volume is $\frac{\sqrt{3}}{2} \cdot \frac{3}{4} = \frac{3\sqrt{3}}{8}$.



- 24) (D) $\arccos(-5/13)$ is the 4th quadrant angle whose cosine is -5/13. Therefore, the sine of this angle is 12/13.

- 25) (D) Let t be the time (in hours) that the candle burns. Then the amount that is burned off of the first candle in t hours is t/6 (so 1-t/6 remains) and the amount burned off of the second candle is t/3 (or 1-t/3 remains). The question asks for the time t when $1 - t/6 = 2(1 - t/3)$. The solution is t = 2 hours.

