2003 MATH OLYMPICS

LEVEL I

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1. $\sqrt{\dfrac{1}{9} + \dfrac{1}{16}} =$

    a. $\dfrac{1}{5}$
    b. $\dfrac{2}{7}$
    c. $\dfrac{5}{12}$
    d. $\dfrac{7}{12}$
    e. None of the above

2. Which of these numbers is largest?

    a. $\sqrt{\sqrt[3]{5 \cdot 6}}$
    b. $\sqrt{6\sqrt[3]{5}}$
    c. $\sqrt{5\sqrt[3]{6}}$
    d. $\sqrt[3]{5\sqrt{6}}$
    e. $\sqrt[3]{6\sqrt{5}}$

3. Last year a bicycle cost $160 and a cycling helmet cost $40. This year the cost of the bicycle increased by 5% and the cost of the helmet increased by 10%. The percent increase in the combined cost of the bicycle and the helmet is

    a. 6%
    b. 7%
    c. 7.5%
    d. 8%
    e. None of the above

4. A vacuum pump removes ½ of the air in a container with each stroke. After 5 strokes, the percentage of the original amount of air that remains in the container will be

    a. 1/2%
    b. 1/32%
    c. 3.125%
    d. 1/8%
    e. None of the above

5. The ratio of w to x is 4:3, of y to z is 3:2, and of z to x is 1:6. What is the ratio of w to y?

    a. 1:3
    b. 16:3
    c. 20:3
    d. 12:1
    e. None of the above

6. Find the difference of $\dfrac{1}{x+1}$ and $\dfrac{x-1}{x^2-1}$.

    a. No difference because both are undefined at -1 b. $\dfrac{-2}{x^2-1}$

    c. 0   d. $\dfrac{1}{x-1}$ e. None of the above

7. The number of real solutions of the equation $|x-2|+|x-3| = 1$ is

    a. 0   b. 1   c. 2   d. 3   e. More than 3

8. The sum of the solutions to $x^2 - x = 6$ is

    a. 1   b. -1 c. -5 d. 13   e. None of the above

9. If $f(x) = 1 - x^2$, find a constant c so that $\dfrac{f(a+h) - f(a)}{h} = c(2a+h)$.

    a. $c=2$ b. $c=-2$ c. $c=1$ d. $c=-1$ e. None of the above

10. If $m>0$ and the points $(m,3)$ and $(1,m)$ lie on a line with slope m, then $m=$

    a. 1   b. $\sqrt{2}$ c. $\sqrt{3}$ d. 2   e. None of the above

11. For how many integers n between 1 and 100 does x^2+x-n factor into the product of two linear factors with integer coefficients?

    a. 8   b. 9   c. 10   d. 12   e. None of the above

12. For the triangle formed by the points $A(-3,2)$, $B(5,4)$, and $C(3,-8)$, write the equation of the line that contains the altitude of the triangle through point C in the form $y=mx+b$.

   a. $y = \dfrac{1}{4}x + 4$ b. $y = -\dfrac{1}{4}x - 7\dfrac{1}{4}$ c. $y = -4x + 4$

   d. $y = -4x + 3$ e. None of the above

13. A parent is currently 3 times as old as his/her child; and in 10 years he/she will be twice as old as his/her child. How many years older is the parent than the child now?

   a. 15 years b. 25 years c. 35 years d. 45 years e. None of the above

14. The following system of equations has only one solution if

$$\begin{cases} kx + y = 1 \\ x + ky = 1 \end{cases}$$

   a. $k=1$ b. $k=0$ c. $k \geq 0$ d. $k \neq \pm 1$ e. None of the above

15. The concentration of a mixture consisting of 10 gallons 20%-acid and 40 gallons 15%-acid is

   a. 12%-acid b. 16%-acid c. 17%-acid d. 15%-acid e. None of the above

16. What is the average of the two solutions of the arbitrary quadratic equation $ax^2+bx+c=0$?

   a. $\dfrac{b}{2a}$ b. $-\dfrac{b}{2a}$ c. $-\dfrac{c}{2a}$ d. $-\dfrac{b}{a}$ e. None of the above

17. A line passes through $(5, -6)$. Which of the following are possible values of the slope m of the line, if the line never enters the first quadrant?

    a. $m \le \dfrac{-6}{5}$ b. $m \le \dfrac{-5}{6}$ c. $\dfrac{-5}{6} \le m \le 0$

    d. $\dfrac{-6}{5} \le m \le 0$ e. None of the above

18. Bill scores 78 on a test that had 4 problems worth 7 points each and 24 multiple-choice questions worth 3 points each. If he had one of the 7-point problems wrong, how many of the multiple-choice questions did he miss?

    a. 3   b. 4   c. 5   d. 6   e. None of the above

19. In $\triangle ABC$, $\angle A = 55°$, $\angle C = 75°$, D is on side $\overline{AB}$ and E is on side $\overline{BC}$. If $DB=BE$, then $\angle BED=$

    a. 50°   b. 55°   c. 60°
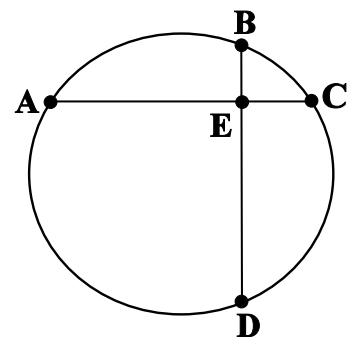
    d. 65°   e. None of the above
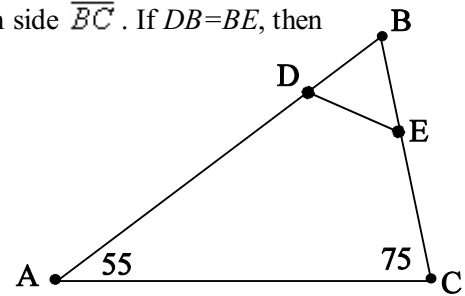
20. If $\angle A$ is four times $\angle B$, and the complement of $\angle B$ is four times the complement of $\angle A$, then $\angle B=$

    a. 10°   b. 12°   c. 15°   d. 18°   e. None of the above

21. The points A, B, C, and D lie on the circle. If $\overline{AC} \perp \overline{BD}$, DE=2, BE=1, and EC=1/2, what is the length of $\overline{AB}$?

    a. $\sqrt{17}$ b. 4   c. $\sqrt{5}$

    d. 3   e. None of the above

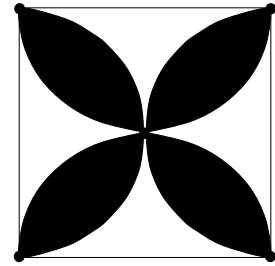22. The area of the shaded region formed by the four semicircles in the given unit square is

    a. $\frac{\pi}{2} - 1 \ \text{cm}^2$ b. $1 - \frac{\pi}{6} \ \text{cm}^2$ c. $1 - \frac{\pi}{8} \ \text{cm}^2$

    d. $\frac{\pi}{4} \ \text{cm}^2$ e. None of the above

1 cm

23. In how many ways can 6 people be lined up to get on a bus if 3 specific persons insist on following each other?

    a. 144   b. 124   c. 24   d. 720   e. None of the above

24. Assume that girl–boy births are equally probable. The probability that a family with 5 children has at least one girl is

    a. 1/5   b. 31/32   c. 4/5   d. 4/32   e. None of the above

25. An old car has to travel a 2–mile route, 1 mile uphill and 1 mile downhill. Because it is so old, the car can climb the first mile—the ascent—no faster than an average speed of 15 mi/hr. How fast does the car need to travel the second mile—on the descent it can go faster, of course—in order to achieve an average speed of 30 mi/hr for the trip?

    a. 45 mi/hr   b. 60 mi/hr   c. 75 mi/hr

    d. 100 mi/hr   e. None of the above