2002 MATH OLYMPICS

LEVEL I

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1. The expression $\left(3^{-1} - 2^{-1}\right)^{-1}$ is equal to

    a. 1 b. $-1/6$ c. -1 d. $1/6$ e. None of the above

2. The width of a rectangle is one foot more than its length. Determine the perimeter of the rectangle, in feet, if the area of the rectangle is one square foot.

    a. $\sqrt{5}$ b. $2\sqrt{5}$ c. $\dfrac{\sqrt{5}-1}{2}$ d. $\sqrt{5}-1$ e. None of the above

3. For the function $f(x) = ax^2 + bx + c,$ where $f(0) = 1,$ $f(1) = 3,$ and $f(2) = 2$, what is a?

    a. -2 b. $-3/2$ c. $3/2$ d. $7/2$ e. None of the above

4. An equilateral triangle is inscribed in a circle with a radius of 2 inches. Determine the length of each of the sides of the equilateral triangle.

    a. $2\sqrt{2}$ b. $2\sqrt{3}$ c. 4 d. 2 e. None of the above

5. The mean (average) score on a test is 75. A grading error is discovered on one student's test, and the student's score is increased by 10 points. If the revised mean is 75.4, how many students took the test?

    a. 25 b. 30 c. 40 d. 50 e. None of the above

6. When choosing three numbers from the set $\{0,1,2,3,4,5,6,7,8,9\}$ at random, with repetition allowed, what is the probability that precisely 2 of these will be the same?

    a. 1/10 b. 3/10 c. 1/3 d. 9/100 e. None of the above

7. Triangle BCD is inscribed in a semi-circle of diameter 4 as shown in the figure. The area A of triangle BCD as a function of x is
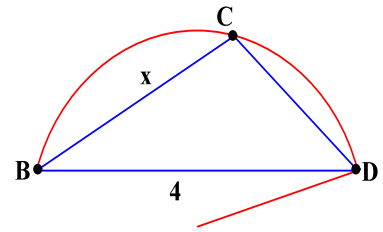
a. $A = \dfrac{x\sqrt{16 - x^2}}{2}$

b. $A = \dfrac{x\sqrt{4 - x^2}}{2}$

c. $A = \dfrac{xy}{2}$

d. x

e. None of the above

8. The expression $\log_{1/9} 3$ is equal to

a. ½   b. 2   c. – ½   d. 1/3   e. None of the above

9. For the function $f(x) = -2x^2$, $(f \circ f)(1)$ is equal to

a. – 2   b. – 8   c. – 4   d. 8   e. None of the above

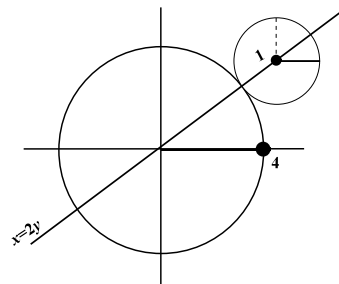10. The center of the small circle as shown in the figure is the point

a. $\left(2\sqrt{5}, \sqrt{5}\right)$ b. $\left(5, \tfrac{5}{2}\right)$ c. (6,3)

d. (4,2)   e. None of the above

11. The expression $\left(\dfrac{-8x^3}{y^{-6}}\right)^{-1/3}$ can be simplified to

a. $\dfrac{-2x}{y^2}$ b. $\dfrac{x}{2y^2}$ c. $\dfrac{-2}{xy^2}$ d. $\dfrac{-1}{2xy^2}$ e. None of the above

12. A woman begins jogging at 3:00 p.m., running due north at 10 mph. Later she reverses direction and runs due south at 12 mph. If she runs back to her starting point at 3:44 p.m., the total number of miles she ran is

   a.   11 miles   b.   8.5 miles   c.   8 miles   d.   7.5 miles e.   None of the above

13. The last digit of 2^{2002} is

   a.   2   b.   4   c.   6   d.   8   e.   None of the above

14. In the figure, $ABCD$ is a rectangle and B is a midpoint of the larger rectangle. What is the area of $ABCD$?

   a.   18   b. $18\sqrt{2}$ c.   36
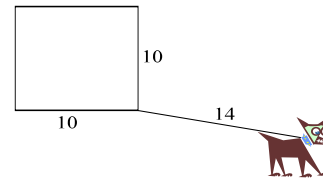
   d. $36\sqrt{2}$ e.   None of the above
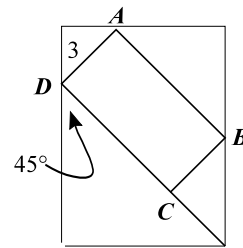
15. Exactly one of the following statements is false:

   1)   Audrey is older than Beatrice.
   2)   Clement is younger than Beatrice.
   3)   The sum of the ages of Beatrice and Clement is twice the age of Audrey.
   4)   Clement is older than Audrey.

   Who is the youngest?

   a.   Audrey   b.   Beatrice   c.   Clement

   d.   They are all the same age   e.   There is not enough information given

16. The digits 1, 3, 4, 6, 8, and 9 are each used exactly once to form three two-digit prime numbers p, q and r with $p>q>r$. What is p–q+r?

   a.   55   b.   63   c.   67   d.   71   e.   None of the above

17. The point (a,b) is reflected over the y-axis to the point (c,d), which is reflected over the x-axis to the point (e,f). Compute ab–ef.

   a.   0   b.   2a   c.   2b   d.   2c   e.   None of the above

18. A dog is tethered to the corner of a ten-foot-by-ten-foot square pen by a leash that is fourteen feet long. In how much area outside the pen can the dog roam?

    a. 147π b. 151π c. 155π

    d. 159π e. None of the above

19. The sum of all different real values of x for which $\left(x^2 - 5x + 5\right)^{x^2-9x+20} = 1$ is

    a. 9   b. 10   c. 12   d. 15   e. None of the above

20. All Sufs are Tufs. One third of all Tufs are Sufs. Half of all Rufs are Tufs. One Ruf is a Suf. Eight Rufs are Tufs. The number of Tufs is 90. How many Tufs are neither Sufs or Rufs?

    a. 52   b. 53   c. 45   d. 46   e. None of the above

21. Find the distance from the point $(8, -1)$ to the line described by $y = \tfrac{1}{2}x + 5$.

    a. 2   b. 5   c. $4\sqrt{5}$ d. $\sqrt{213}$ e. None of the above

22. The graphs of $y = -\dfrac{1}{2}x^2 + 2$ and $x^2 + y^2 = 4$ have how many points in common?

    a. 0   b. 2   c. 3   d. 4   e. None of the above

23. If 3 apples and 2 oranges cost $0.84 and 6 apples and 1 orange cost $1.32, what is the cost of 2 apples and 2 oranges?

    a. $0.20   b. $0.12   c. $0.42   d. $0.64   e. None of the above

24. Rita travels on an expressway at a rate of 50 mph for 2 hours then at 70 mph for another 3 hours. What is her average speed for the entire trip?

    a. 58 mph   b. 60 mph   c. 62 mph   d. 63 mph   e. None of the above

25. A square and a circle have equal perimeters. What is the ratio of the side of the square to the radius of the circle?

   a. 2:1   b. 2:π c. π:2   d. 1:2π e. None of the above