Saginaw Valley State University

2022 Math Olympics — Level I

1. Consider the equation $p(x) : ax^2 + bx + c = 0$ whose coefficients a, b and c are all non-zero, and each of them satisfies an equation that results from removing the term containing that coefficent from the equation $p(x)$; for example, the coefficient b is a solution of the equation $ax^2 + c = 0$. What is the sum of all solutions of $p(x)$?

   (a) Always 1 **(b)** Always -1 **(c)** Always 2 **(d)** 1 or -1 **(e)** 1 or 2

2. A marching band has 150 members. One day only part of them show up, but nobody wants to take the time to count how many there are. They first line up in rows with 5 members each, but there is one left over. Then they try rows with 6 members each, but there is still one left over. Then they try rows of 7, but there are two left over. How many people should line up in each row so no members will be left over?

   (a) 4 **(b)** 11 **(c)** 13 **(d)** 17 **(e)** None of the above

3. Which one of the following is the only true statement?

   (a) The graph of a horizontal line can't have any x-intercepts.

   (b) The graph of a horizontal line can't have a unique x-intercept, but may have more than one x-intercept.

   (c) The graph of a parabola can't have a unique x-intercept, but may have more than one x-intercept.

   (d) The graph of a polynomial of degree three or higher must have at least one x-intercept.

   (e) Either none are true or more than one are true.

4. Which of the following is equal to 10!? (If you are not familiar with the notation, the number $n!$ is defined for any non-negative integer using the following recursive formula:

$$0! = 1$$
$$n! = n \cdot ((n-1)!) \text{ for } n > 0)$$

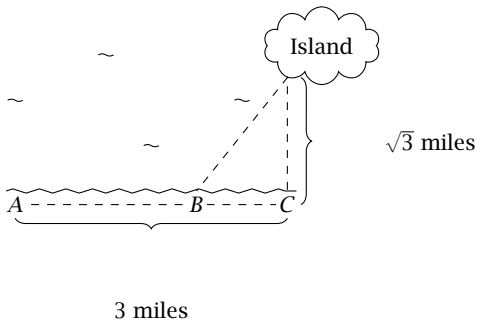
   (a) $5! \cdot 2!$ **(b)** $7! \cdot 5! \cdot 3!$ **(c)** $7! \cdot 5! \cdot 2!$ **(d)** $7! \cdot 5! \cdot 3! \cdot 2!$

   (e) None of the above or more than one of the above.

5. The base 6 expansion of one number is 550. The base 5 expansion of another number is 3440. What is their greatest common divisor expanded in base 4?

   (a) 24 **(b)** 33 **(c)** 113 **(d)** 223 **(e)** None of the above

6. A person walks along a beach, starting at point A, at a rate of 3 mi/h and at point B, goes into the water and swims at a rate of 2 mi/h diagonally out to an island that is a distance of $\sqrt{3}$ mi from point C, directly across from the island on the shore, as shown in the picture. The total distance from point A to point C is 3 mi. There are two different choices for the distance, in miles, from point A to point B that will result in a total time for walking and swimming of one hour and 40 minutes; what is the sum of those numbers?



(a) 2 **(b)** 4 **(c)** $\frac{14}{5}$ **(d)** $\frac{16}{5}$ **(e)** None of the above

7. Which of the following is equal to $|\sqrt{2022} - 45|$?

(a) $\sqrt{3}$ **(b)** $\sqrt{1977}$ **(c)** $\sqrt{2022} - 45$ **(d)** $\sqrt{2022} + 45$

(e) $45 - \sqrt{2022}$

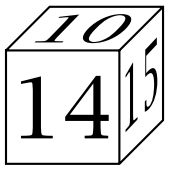
8. Which of the following is equal to $\sqrt[6]{s^5}\sqrt[9]{s}$ for all non-negative values of s?

(a) $\sqrt[3]{s}$ **(b)** $\sqrt[5]{s^2}$ **(c)** $\sqrt[54]{s^5}$ **(d)** $\sqrt[18]{s^{17}}$ **(e)** None of the above

9. If 7^3 cubes are stacked to form a $7 \times 7 \times 7$ cube, how many small cubes are on the surface of the large cube?

(a) 127 **(b)** 134 **(c)** 218 **(d)** 294 **(e)** 327

10. Each side of the cube depicted on the right is numbered with a positive integer in such a way that the products of the numbers on each pair of opposite sides are all the same. Find the lowest possible sum of all the numbers on the sides of the cube.



(a) 78 **(b)** 80 **(c)** 89 **(d)** 107 **(e)** None of the above

11. Find the largest integer smaller than $\sqrt{22 + \sqrt{22 + \sqrt{22 + \sqrt{22}}}}$.

(a) 4 **(b)** 5 **(c)** 9 **(d)** 20 **(e)** 25

12. Pat and Mat were trying to calculate the average of two numbers a and b using their calculator. First, Pat took the calculator and typed in $a + b \div 2$ and got 30. Then Mat took the same calculator and typed in $b + a \div 2$ and got 18. What was the correct average of the two numbers?

(a) 28 **(b)** 24 **(c)** 16 **(d)** 12 **(e)** None of the above

13. How many pairs of integers (x, y) are solutions of the equation

$$3x^2y - 10xy - 8y - 17 = 0?$$

(a) none **(b)** one **(c)** two **(d)** four **(e)** None of the above

14. For how many integers is $\dfrac{11n + 14}{n - 2}$ an integer?

(a) 3 **(b)** 6 **(c)** 9 **(d)** 18 **(e)** None of the above

15. A fair coin is tossed 100 times and it lands on heads all 100 times. What is the probability that it will land on heads on the 101st toss?

(a) 1 **(b)** $\frac{1}{2^{100}}$ **(c)** $\frac{1}{2^{101}}$ **(d)** $\frac{1}{2}$ **(e)** None of the above

16. If two woodchucks would chuck 32 lb of wood in 12 min, how much wood would 3 woodchucks chuck in 8 min?

(a) $21\frac{1}{3}$ lb **(b)** 32 lb **(c)** 64 lb **(d)** 80 lb **(e)** None of the above

17. Assuming that 'wigglers' are those who wiggle, 'wobblers' are those that wobble, and 'wagglers' are those who waggle, which of the following sets of premises will necessarily lead to the conclusion that "Wilbur is not a weeble"?

(a) All weebles wobble.
No wobblers wiggle.
Some wigglers waggle.
Wilbur waggles.

(b) Some weebles wobble.
No wobblers wiggle.
Some wagglers wiggle.
Wilbur waggles.

(c) All weebles wobble.
No wobblers wiggle.
All wigglers waggle.
Wilbur waggles.

(d) All weebles wobble.
No wobblers wiggle.
All wagglers wiggle.
Wilbur waggles.

(e) None of the above

18. Which of the expressions is equal to 0 for every x?

(a) $(x + 1)(x - 1) - x^2 + 1$ **(b)** $x^0 - 1$ **(c)** $\sqrt{x^2} - x$

(d) None of them is equal to 0 for every x. **(e)** More than one of them is 0 for every x.

19. The vertices of an equilateral triangle lie on a circle with radius 2. The area of the triangle is

(a) $3\sqrt{3}$ **(b)** $2\sqrt{3}$ **(c)** $5\sqrt{3}$ **(d)** $4\sqrt{3}$ **(e)** None of the above

20. How many 7-digit positive integers are made up of the digits 0 and 1 only and are divisible by 6?

    (a) 10 **(b)** 11 **(c)** 16 **(d)** 21 **(e)** 33

21. Suppose that x and y satisfy $\frac{x-y}{x+y} = 9$ and $\frac{xy}{x+y} = -60$. The value of $(x+y)+(x-y)+xy$ is

    (a) -50 **(b)** -150 **(c)** -14310 **(d)** 210 **(e)** 14160

22. A total of n points are equally spaced around a circle and are labeled with the integers 1 to n, in order. Two points are called diametrically opposite if the line segment joining them is a diameter of the circle. If the points labeled 7 and 35 are diametrically opposite, then n equals

    (a) 54 **(b)** 55 **(c)** 56 **(d)** 57 **(e)** None of the above

23. What is the area of a rhombus that has sides of length 10 cm and diagonals that differ in length by 4 cm?

    (a) $96\,\text{cm}^2$ **(b)** $100\,\text{cm}^2$

    (c) $100\sqrt{2}\,\text{cm}^2$ **(d)** Not enough information given.

    (e) None of the above

24. Point P lies inside an equilateral triangle whose sides are of length 2. If the distances from P to each side of the triangle are x, y and z, what is $x+y+z$?

    (a) $\sqrt{3}$ **(b)** $2\sqrt{3}$

    (c) $3\sqrt{3}$ **(d)** Not enough information given.

    (e) None of the above

25. A rectangle is given with length l and width w, where $l > w$. One of them is increased by 20%, while the other is decreased by 20%. What happens to the area of the rectangle?

    (a) It stays the same. **(b)** It always decreases.

    (c) It always increases. **(d)** It increases only if the l is increased.

    (e) It increases only if the w is increased.