Saginaw Valley State University

2019 Math Olympics — Level I Solutions

1. Jacob wanted to get rid of most of his toy cars. He gave one-sixth of his collection, plus 8 cars, to Xu, and one-fourth of what was left, plus 4 cars, to Olivia. Then he gave half of what was left to Ahmed. Then he gave 40% of what was left, plus 5 cars, to Jim. He kept the last 10 cars. How many toy cars did Jacob have at the beginning?

   (a) 116 **(b)** 106 **(c)** 96 **(d)** 86 **(e)** None of the above

   SOLUTION **(c)**: Let x be the number of cars Jacob had at the beginning. After he gave one-sixth of x, plus 8 cars, to Xu, he had

$$\frac{5}{6}x - 8$$

cars left. After he gave one-fourth of that, plus 4 cars, to Olivia, he had

$$\frac{3}{4}\left(\frac{5}{6}x - 8\right) - 4$$

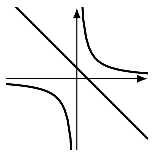
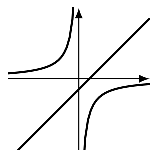
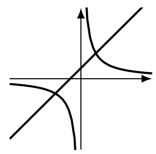
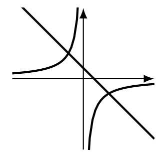
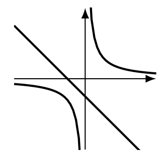
cars left, and after giving half of that to Ahmed and 40% (or two-fifths) of what was left, plus 5 cars, to Jim, he had

$$\frac{3}{5}\left(\frac{1}{2}\left(\frac{3}{4}\left(\frac{5}{6}x - 8\right) - 4\right)\right) - 5$$

cars left. We know that this last number was 10. Therefore

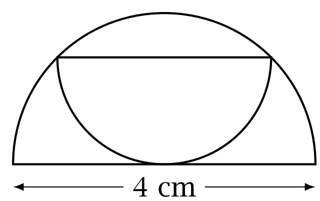
$$\begin{aligned}
\frac{3}{5}\left(\frac{1}{2}\left(\frac{3}{4}\left(\frac{5}{6}x - 8\right) - 4\right)\right) - 5 &= 10 \\
\frac{3}{10}\left(\frac{3}{4}\left(\frac{5}{6}x - 8\right) - 4\right) &= 15 \\
\frac{3}{4}\left(\frac{5}{6}x - 8\right) - 4 &= 50 \\
\frac{5}{8}x - 6 &= 54 \\
\frac{5}{8}x &= 60 \\
x &= 96
\end{aligned}$$

2. Let k be a non-zero real number. Two functions, $f(x) = \frac{k}{x}$ and $g(x) = kx + k$, were plotted on the same coordinate plane. Which of the following is the correct plot?

(a)  **(b)**  **(c)**  **(d)**  **(e)** 

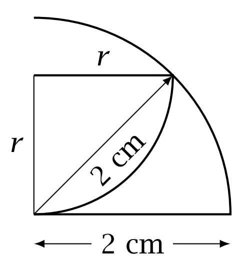
SOLUTION **(c)**: We can eliminate all the incorect choices:
In choices **(a)** and **(d)**, the line has negative slope but positive y-intercept, so it cannot be of the form $y = kx + k$.
In choice **(b)**, the line has positive slope but negative y-intercept.
In choice **(e)**, the line has negative slope and y-intercept, so it could be of the form $y = kx + k$ for a negative k, but the hyperbola is of the form $y = \frac{k}{x}$ for a positive k.
The choice **(c)** is the only one left, and shows a line with a positive slope and y-intercept, and a hyperbola with a positive coefficient k.

3. A small semi-circle is inscribed in a large semi-circle as shown on the right. What is the radius of the small semi-circle?



(a) $\frac{\sqrt{2}}{2}$ cm **(b)** $\sqrt{2}$ cm **(c)** 1.5 cm **(d)** 2 cm

(e) None of the above



SOLUTION **(b)**: Because of symmetry, the center of the small semi-circle must be directly above the center of the large semi-circle. The right half of the picture is shown on the left. Here r is the radius of the small semi-circle. The two radii shown form the arms of an isosceles right triangle with hypotenuse 2 cm. From the Pythagorean Theorem, it follows that $2r^2 = 4\text{ cm}^2$, or $r = \sqrt{2}$ cm.

4. Suppose you know that:

   ⊢ If thistles whistle, then weebles wobble.
   ⊢ Ants dance only if goggles giggle.
   ⊢ If thistles don't whistle, then ants dance.

   Which of the following must logically be true?

   (a) If weebles wobble, then goggles giggle.

   (b) Weebles don't wobble only if goggles giggle.

   (c) If ants dance, then weebles wobble. **(d)** Goggles giggle only if thistles whistle.

   (e) None of the above

   SOLUTION **(b)**: Let w = weebles wobble, r = thistles whistle, p = ants dance, and q = goggles giggle. We are thus given:

$$r \to w$$
$$\sim q \to \sim p$$
$$\sim r \to p$$

The first premise $r \to w$ is equivalent to its contraposition: $\sim w \to \sim r$, so it can be replaced. Similarly, the second premise can be replaced by $p \to q$. Replacing and reordering, we get

$$\sim w \to \sim r$$
$$\sim r \to p$$
$$p \to q$$

which is equivalent by transitivity to $\sim w \to q$.
This can be translated to 'If weebles don't wobble, then goggles giggle.', which is equivalent to 'Weebles don't wobble only if goggles giggle.'

5. To simplify your life, you decide to only buy red socks and green socks. This worked so well that you were able to devote your time to other matters, and soon you can afford to hire a housekeeper from the cheapest agency in town. You and your brother come home one day, and the tired housekeeper tells you that he tried to organize your socks for you, but he couldn't fit all the red socks in one drawer or all the green socks in one drawer. Thus, he filled one drawer with red socks, one with green socks, and placed the rest of both red and green socks in another drawer. He labeled the top drawer of your three-drawer dresser 'red only', the middle one 'green only', and the bottom one 'both'. The housekeeper then asks if anyone has seen his glasses before trudging home. Your brother checks the drawers and bursts out laughing. He tells you that although the housekeeper did sort the socks as described, not a single drawer is labeled correctly. He then bets you $100 that you can't correctly relabel the drawers after choosing just one of the drawers and having him hand you just one sock out of that drawer. You need the money so that you can hire a new housekeeper! Which of the following is true?

   (a) The only way to correctly relabel the drawers is to choose the top drawer (labeled 'red only').

   (b) The only way to correctly relabel the drawers is to choose the middle drawer (labeled 'green only').

   (c) The only way to correctly relabel the drawers is to choose the bottom drawer (labeled 'both').

   (d) You can correctly relabel the drawers no matter which drawer you pick.

   (e) You should not take the bet because, no matter which drawer you pick, you will not have enough information to correctly relabel the drawers.

   SOLUTION **(c)**: In the table below, R=red, B = both, G = green.
   Work from the bottom. The bottom row can't contain both colors, since you know the label is wrong. Thus, the bottom drawer contains either red socks only or green socks only.

Drawer	**Existing Labels**	**Possible contents**	
TOP	R		
MIDDLE	G		
BOTTOM	B	R	G

   Since you know the other labels are wrong, there is only one way to complete each column.

Drawer	**Existing Labels**	**Possible contents**	
TOP	R	G	B
MIDDLE	G	B	R
BOTTOM	B	R	G

   If you pick the top drawer, you know it will contain either both colors or only green socks. If the selected sock is green, you won't be able to decide which case is true.

If you pick the middle drawer, you know it will either contain only red socks or both colors. If the selected sock is red, you won't be able to tell which case is true.
However, if you pick the bottom drawer, you know it either contains only red socks or only green socks, and whichever color sock is selected is the color of the socks in that drawer. So picking the bottom drawer is the only way to make sure you can correctly re-label the drawers.

6. Which of the following numbers is different from the rest?

(a) $\dfrac{1+\sqrt{5}}{2}$ **(b)** $\dfrac{(1+\sqrt{5})^2}{4} - 1$ **(c)** $\sqrt{\dfrac{3+\sqrt{5}}{2}}$

(d) $\dfrac{2}{\sqrt{5}-1}$ **(e)** They are all the same.

SOLUTION **(e)**: Let

$$a = \frac{1+\sqrt{5}}{2}$$

Using the quadratic formula you can see that a is a solutuion to the equation

$$x^2 - x - 1 = 0$$

so

$$a = a^2 - 1 = \frac{(1+\sqrt{5})^2}{4} - 1$$

Since

$$a^2 = \frac{(1+\sqrt{5})^2}{4} = \frac{1+2\sqrt{5}+5}{4} = \frac{3+\sqrt{5}}{2}$$

and since a is positive,

$$a = \sqrt{\frac{3+\sqrt{5}}{2}}$$

Finally

$$\frac{2}{\sqrt{5}-1} = \frac{2}{\sqrt{5}-1}\frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{2\,(1+\sqrt{5})}{4} = a$$

All the numbers are equal.

7. Find the area of the triangle bounded by the x-axis and the lines $2y = 3x$ and $y = -3x + 9$.

   (a) 4.5 **(b)** 3 **(c)** 13.5 **(d)** 9 **(e)** None of the above

   SOLUTION **(a)**: First find the intersection points. The line $2y = 3x$ intersects the x-axis at $(0, 0)$. The line $y = -3x + 9$ intersects the x-axis at $(3, 0)$. So the length of the base is 3. The intersection point of the two lines has y-coordinate 3: Solve

$$\begin{cases} 2y = 3x \\ \ \ y = -3x + 9 \end{cases}$$

   Rewrite this as

$$\begin{cases} -3x + 2y = 0 \\ \quad 3x + y = 9 \end{cases}$$

   Adding the equations together gives $3y = 9$ or $y = 3$. Therefore height is 3. So the area is $\frac{1}{2}3 \cdot 3 = 4.5$.

8. Let z and w be complex numbers, and for any complex number x, let $\overline{x}$ denote the complex conjugate of x. Which of the following must be a real number?

   (a) $z(\overline{z} + w) + \overline{zw}$ **(b)** $z\overline{w} - \overline{z}w$ **(c)** $z + \overline{\overline{z}}$ **(d)** $(z - \overline{z})(z + \overline{z})$

   (e) Either more than one or none of them are real numbers.

   SOLUTION **(a)**: Using the fact that for each complex number ζ, both $\zeta \cdot \overline{\zeta}$ and $\zeta + \overline{\zeta}$ are real numbers, and rewriting $z(\overline{z} + w) + \overline{zw}$ as $z \cdot \overline{z} + zw + \overline{zw}$, we get a sum of two real numbers which is again a real number.
   On the other hand, setting for example $z = i$ and $w = 1$, we get $z\overline{w} - \overline{z}w = i \cdot 1 - (-1) \cdot 1 = 2i$ which is not a real number, and, similarly, $z + \overline{\overline{z}} = i + \overline{-i} = i + i = 2i$.
   Finally, setting $z = 1 + i$ we get $(z - \overline{z})(z + \overline{z}) = ((1 + i) - (1 - i))((1 + i) + (1 - i)) = (2i)(2) = 4i$ which is not a real number.

9. Which of the following is the domain of the function

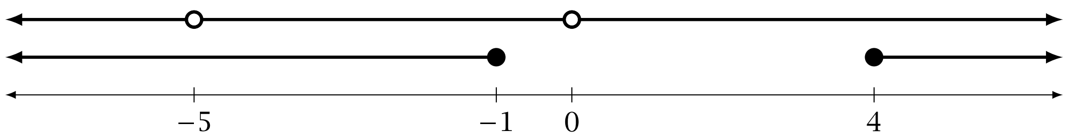
$$f(x) = \frac{\sqrt{x^2 - 3x - 4}}{\sqrt[3]{x(x+5)}} \ ?$$

(a) $(-\infty, -5) \cup [-1, 0) \cup [4, \infty)$ **(b)** $(-\infty, -5) \cup (-1, 0) \cup (4, \infty)$

(c) $(-\infty, -5) \cup (-5, -1] \cup [4, \infty)$ **(d)** $(-\infty, -5) \cup [4, \infty)$

(e) None of the above

Solution **(c)**: In order for $f(x)$ to be defined, $x^2 - 3x - 4 \geq 0$ and $x(x+5) \neq 0$. Factoring will turn the first inequality into $(x-4)(x+1) \geq 0$, or $x \in (-\infty, -1] \cup [4, \infty)$.
The second inequality is equivalent to $x \neq -5$ and $x \neq 0$.



We can see that if both inequalities are to be satisfied, x must be in $(-\infty, -5) \cup (-5, -1] \cup [4, \infty)$.

10. Solve the inequality $\dfrac{1}{x-3} \leq 1$

(a) $(-\infty, 3) \cup [4, \infty)$ **(b)** $(-\infty, 4]$ **(c)** $[4, \infty)$

(d) $(3, 4]$ **(e)** None of the above

Solution **(a)**: Subtracting 1 from both sides gives us

$$\frac{1}{x-3} - 1 \leq 0$$
$$\frac{1}{x-3} - \frac{x-3}{x-3} \leq 0$$
$$\frac{1-(x-3)}{x-3} \leq 0$$
$$\frac{4-x}{x-3} \leq 0$$

We see that the numerator contains the factor $4 - x$, which is linear with negative slope, and x-intercept at 4. The denominator then contains the factor $x - 3$, which is linear, with positive slope, and x-intercept at 3. We can construct the following sign-chart:

$4-x$	+		+	0	−
$x-3$	−	0	+		+
$\frac{4-x}{x-3}$	−	×	+	0	−
		3		4	

Since we are looking for areas where $\frac{4-x}{x-3}$ is less than or equal to 0, we get $(-\infty, 3) \cup [4, \infty)$.

11. Let f be a function and let $g(x) = 2\,|f(3-x)| - 1$. Suppose that $(1, -5)$ is a point on the graph of f. Which of the following points must be on the graph of g?

(a) $(2, 9)$ **(b)** $(2, 12)$ **(c)** $(-4, 9)$ **(d)** $(-4, 12)$ **(e)** None of the above

SOLUTION **(a)**: Since $(1, -5)$ is on the graph of f, we know that $f(1) = -5$. Since we do not have any other information about f, the only way we can find the value of $g(x)$ is if $3 - x = 1$, or $x = 2$. Then $g(2) = 2\,|f(1)| - 1 = 2\,|-5| - 1 = 9$, so the point $(2, 9)$ is on the graph of g.

12. If 3½ children can make 2½ snowmen in 1½ days, how many days will it take x children to make y snowmen?

(a) $\dfrac{5y}{7x}$ **(b)** $\dfrac{10xy}{21}$ **(c)** $\dfrac{10y}{21x}$ **(d)** $\dfrac{21y}{10x}$ **(e)** None of the above

SOLUTION **(d)**: In 1.5 days 1 child can build $\frac{5}{7}$ of a snowman. In one day, 1 child can build $\frac{5}{7} \div \frac{3}{2} = \frac{10}{21}$ of a snowman. So x children build $\frac{10}{21}x$ snowmen per day. So it takes $\frac{21}{10x}$ days for x children to build 1 snowman and $\frac{21y}{10x}$ days to build y snowmen.

13. A super amphitheater has 91 seats in the bottom row, 93 seats in the next row, 95 seats in the next row, and so on up to the top. If the total number of seats in the theater is 22,000, how many rows are there?

(a) 220 **(b)** 200 **(c)** 110 **(d)** 260 **(e)** None of the above

SOLUTION **(c)**: The sum of an arithmetic sequence is

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)\,d\right).$$

Here $d = 2$, $a_1 = 91$ and $S_n = 22,000$. Putting in these values and simplifying we get $n^2 + 90n - 22000 = 0$. This factors as $(n-110)(n+200) = 0$. The number of rows must be positive, so $n = 110$.

14. Mike can bike three times as fast as he can run. He enters a two-part biking-running race that is a total distance of 120 miles over a total time of 4 hours. If there is no time break between the two parts, and he can run one mile in 5 minutes, how long did he bike?

(a) 1 hour **(b)** 2 hours **(c)** 3 hours **(d)** 4.5 hours **(e)** None of the above

Solution **(c)**: Call the time spent running t_1 and the time biking t_2. Then $t_1 + t_2 = 4$. His run rate is 12 mi/h ($60/5 = 12$) and so his bike rate is 36 mi/h. Therefore $12t_1 + 36t_2 = 120$. Solve the system

$$\begin{cases} t_1 + t_2 = 4 \\ 12t_1 + 36t_2 = 120 \end{cases}$$

for t_2:

$$\begin{cases} -12t_1 - 12t_2 = -48 \\ 12t_1 + 36t_2 = 120 \end{cases}$$

Adding the equations we get $24t_2 = 72$ and so $t_2 = 3$.

15. In how many different ways can 8 mathematicians and 5 physicists stand in a line if no two physicists can stand next to each other?

(a) $13! - (8!/3!)$ **(b)** $(8! \times 9!)/4!$ **(c)** $8! \times 5! \times {}_8C_5$

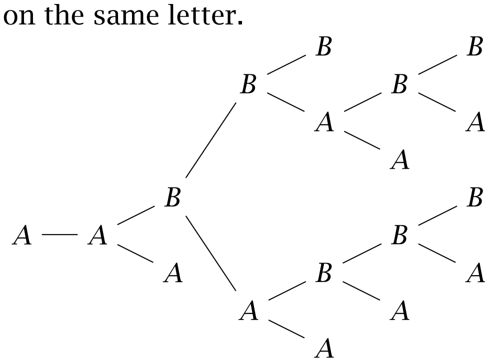
(d) $8! \times {}_9C_5$ **(e)** None of the above

Solution **(b)**: First order the 8 mathematicians. There are ${}_8P_8$ or 8! ways to do that. Then put the physicists in the places between the mathematicians. Since a physicist can go before the first mathematician, between each pair of adjacent mathematicians, and after the last mathematician, there are 9 places a physicist can go, so there are ${}_9P_5$ or $9!/4!$ ways to place the physicists.

16. A tennis tournament is played between 2 teams, team A and team B. A winner is declared if a team wins three games in a row or four games. If we assume no ties, and we know that team A has won the first two games, how many different ways are there for the tournament to finish? (For example, A wins, A wins, B wins B wins A wins A wins is one way, since team A has won 4 times.)

    (a) 9 **(b)** 16 **(c)** 32 **(d)** 8 **(e)** None of the above

    SOLUTION **(a)**: We can make a tree diagram showing all the possible situations: A will mean the team A won the game, B will mean the team B won the game. Each branch will be one possible scenario. Each branch will end if either it has three of the same letters in a row, or 4 on the same letter.



17. Which of the following functions is not even?

    (a) $f(x) = (x^2 - 1)^3$ **(b)** $g(x) = (x^3 - x)^2$ **(c)** $h(x) = (x^3 - 1)^2$

    (d) $k(x) = (x^2 - 1)^2$ **(e)** They are all even.

    SOLUTION **(c)**:

$$f(-x) = ((-x)^2 - 1)^3 = (x^2 - 1)^3 = f(x)$$

so f is even.

$$g(-x) = ((-x)^3 - (-x))^2 = (-x^3 + x)^2 = [-(x^3 - x)]^2 = (x^3 - x)^2 = g(x)$$

so g is even.

$$h(-x) = ((-x)^3 - 1)^2 = (-x^3 - 1)^2 = [-(x^3 + 1)]^2 = (x^3 + 1)^2 \neq h(x)$$

so h is not even. At this moment we can stop, but just for completeness:

$$k(-x) = ((-x)^2 - 1)^2 = (x^2 - 1)^2 = k(x)$$

so k is even.

18. Let $x \oplus y$ be defined by $x \oplus y = \dfrac{x}{y} + \dfrac{y}{x}$ where x and y are non zero real numbers. Which of the following are true?

(a) $\oplus$ is associative but not commutative **(b)** $\oplus$ is commutative but not associative

(c) $\oplus$ is both associative and commutative **(d)** $\oplus$ is neither associative nor commutative

(e) 1 is the identity for $\oplus$

SOLUTION **(b)**: Note that the operation is commutative:

$$y \oplus x = \frac{y}{x} + \frac{x}{y} = \frac{x}{y} + \frac{y}{x} = x \oplus y$$

Also note that $3 \oplus 1 = \frac{1}{3} + \frac{3}{1} \neq 3$, so 1 is not an identity for $\oplus$.
Also note that for a non zero number a, $a \oplus a = 2$, so

$$(3 \oplus 3) \oplus 2 = 2 \oplus 2 = 2$$

while

$$3 \oplus (3 \oplus 2) = 3 \oplus \left(\frac{2}{3} + \frac{3}{2}\right) = 3 \oplus \frac{13}{6} = \frac{18}{13} + \frac{13}{18} = \frac{493}{234} \neq 2$$

so $\oplus$ is not associative.

19. How many positive integers n are there such that $\dfrac{n^3 + 6n + 55}{n - 2}$ is also an integer?

(a) 7 **(b)** 8 **(c)** 9 **(d)** 13 **(e)** There are infinitely many such values of n.

SOLUTION **(a)**: Using synthetic division

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 6 & 55 \\ & & 2 & 4 & 20 \\ \hline & 1 & 2 & 10 & 75 \end{array}$$

we see that

$$\frac{n^3 + 6n + 55}{n - 2} = n^2 + 2n + 10 + \frac{75}{n - 2}$$

so

$$\frac{n^3 + 6n + 55}{n - 2}$$

is an integer if and only 75 is divisible by $n - 2$.

The factors of 75 are ± 1, ± 3, ± 5, ± 15, ± 25 and ± 75, so $n - 2$ must be one of these 12 numbers. However, since n must be positive, $n - 2 \geq -1$, so only 7 of these numbers can be used. Therefore there are 7 positive numbers n such that

$$\frac{n^3 + 6n + 55}{n - 2}$$

is an integer.

20. A mad scientist does all her calculations in a base 26 number system, where the letters of the English alphabet, in the alphabetical order, are used as digits, that is, A represents 0, B represents 1, …, Y represents 24, and Z represents 25.
The full table is below:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	*S*	*T*	*U*	*V*	*W*	*X*	*Y*	*Z*
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

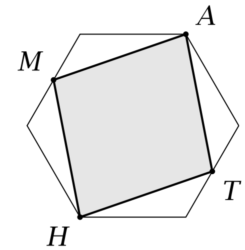
In this system, what is $ONE + TWO$?

(a) *IKS* **(b)** *BIKS* **(c)** *BIJS* **(d)** *BIJE* **(e)** None of the above

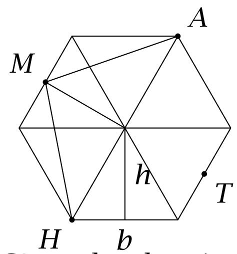
SOLUTION **(c)**:

$$\begin{array}{cccc} & \text{B} & & \\ & \text{O} & \text{N} & \text{E} \\ & \text{T} & \text{W} & \text{O} \\ \hline \text{B} & \text{I} & \text{J} & \text{S} \end{array} \qquad \begin{array}{rrrr} & 1 & & \\ & 14 & 13 & 4 \\ & 19 & 22 & 14 \\ \hline 1 & 8 & 9 & 18 \end{array}$$

21. In the diagram, the points M and T are the midpoints of the opposite sides of a regular hexagon. What fraction of the hexagon is shaded?

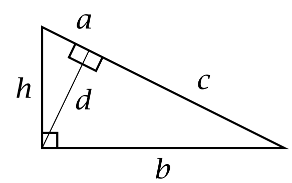


(a) ⅔ **(b)** ¾ **(c)** 5⁄4 **(d)** ⅞ **(e)** 8⁄9



Solution **(a)**: Let b be the length of the side of the regular hexagon. The hexagon the consists of 6 equilateral triangles with side b. Let h be the height of each of these triangles, as shown. The area of each of the triangles is $\frac{1}{2}bh$ so the total area of the hexagon is $3bh$.
The area of $MATH$ is twice the area of $\triangle MAH$ which has the base $2b$ and height h. So the area of $\triangle MAH$ is bh, and the area of $MATH$ is $2bh$. That makes the area of $MATH$ ⅔ of the area of the whole hexagon.

22. Given that $h = 4a$, which of the following does ac equal?



(a) $\frac{1}{2}b^2$ **(b)** $\frac{1}{4}b^2$ **(c)** $\frac{1}{8}b^2$ **(d)** $\frac{1}{16}b^2$

(e) None of the above

Solution **(d)**: According to the right triangle altitude theorem (also known as geometric mean theorem), $ac = d^2$. It is given that $h = 4a$, and from the similarity of triangles, it follows that $b = 4d$, or $d = \frac{b}{4}$. Together

$$ac = d^2 = \left(\frac{b}{4}\right)^2 = \frac{1}{16}b^2$$

23. Suppose a 5-bit codeword is transmitted 5 times, once with 4 errors, once with three errors, once with 2 errors, once with 1 error, and once correctly (not necessarily in that order). The 5 received codewords are given below. Which one is the correct codeword?

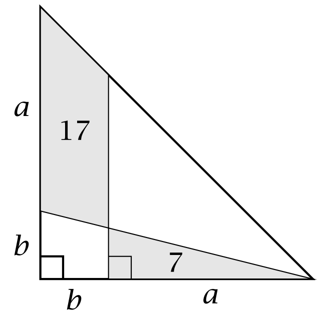
(a) 00101 **(b)** 11010 **(c)** 11100 **(d)** 10100 **(e)** 00001

Solution **(c)**: For each codeword, calculate the Hamming distances (number of bits that differ) from each of the other codewords.

	00101	**11010**	**11100**	**10100**	**00001**
00101	0	5	3	2	1
11010	5	0	2	3	4
11100	3	2	0	1	4
10100	2	3	1	0	3
00001	1	4	4	3	0

The string 11100 is the only one with distances 0, 1, 2, 3, and 4.

24. Two shaded areas in the right triangle are 17 and 7, as marked. Find ab.



(a) 20 **(b)** 24 **(c)** 48 **(d)** $\sqrt{119}$

(e) None of the above

SOLUTION **(a)**: The small trapezoidal area in the lower left corner of the triangle can be written in two different ways:

$$\frac{1}{2}(a + b + a)b - 17$$

or

$$\frac{1}{2}(a + b)b - 7$$

Setting these equal to each other and solving:

$$\begin{aligned} \frac{1}{2}(a + b + a)b - 17 &= \frac{1}{2}(a + b)b - 7 \\ \frac{1}{2}(2a + b)b &= \frac{1}{2}(a + b)b + 10 \\ 2ab + b^2 &= ab + b^2 + 20 \\ ab &= 20 \end{aligned}$$

25. Let $f : \mathbb{R} \to \mathbb{R}$ be an odd periodic function with period 5. Given that $f(7) = 9$, what is $f(2020) - f(2018)$?

(a) 6 **(b)** 7 **(c)** 8 **(d)** 9 **(e)** None of the above

SOLUTION **(d)**: Since the function is periodic with period 5, $f(2020) = f(0)$ and $f(2018) = f(-2)$. Since the function is odd, $f(0) = 0$ and $f(-2) = -f(2) = -9$.
So $f(2020) - f(2018) = 0 - (-9) = 9$.