Saginaw Valley State University

2018 Math Olympics — Level II

1. Which of the following is the value of $\frac{2}{3}+\frac{1}{4}+\frac{2}{15}+\frac{1}{12}+\frac{2}{35}+\frac{1}{24}+\frac{2}{63}\cdots+\frac{2}{(2017)(2019)}$?

   (a) $1-\frac{1}{(2018)(2019)}$ **(b)** $\frac{3}{2}-\frac{4037}{(2018)(2019)}$ **(c)** $1+\frac{1}{(2018)(2019)}$

   (d) $\frac{2018}{2019}$ **(e)** None of the above

2. Function f is defined in the following way:

$$f(0)=1$$
$$f(k)=\frac{f(k-1)}{1+f(k-1)} \text{ for } k\geq 1$$

   What is $f(2018)$?

   (a) $\frac{2017}{2018}$ **(b)** $\frac{1}{2019}$ **(c)** $\frac{2019}{2018}$ **(d)** $\frac{2018}{2019}$ **(e)** None of the above

3. How many positive divisors does $2^2\cdot 3^3\cdot 4^4\cdot 5^5\cdot 6^6\cdot 7^7$ have?

   (a) 5040 **(b)** 20160 **(c)** 8160 **(d)** 7200 **(e)** None of the above

4. $\log_7 5+\log_{49} 3=$

   (a) $\log_7 5\sqrt{3}$ **(b)** $\log_7 45$

   (c) $\log_{49} 75$ **(d)** Both (a) and (c) are correct

   (e) Both (b) and (c) are correct

5. The solution to the equation $2^{x+3}=4\cdot 3^{2x}$ is:

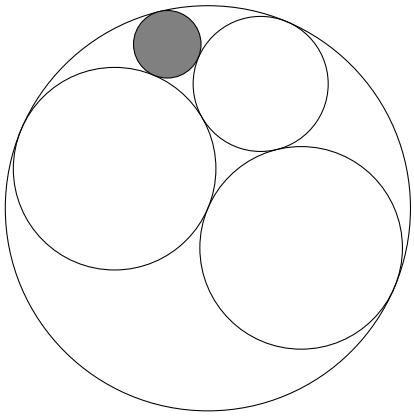
   (a) $\frac{3\ln 2}{2\ln 12-\ln 2}$ **(b)** $\frac{\ln 2}{\ln 4.5}$ **(c)** $\ln\left(\frac{4}{9}\right)$

   (d) $3-\log_2 12$ **(e)** None of the above

6. Two cards are delt from a standard 52 card deck and placed side by side on a table. What is the probability that the first card is a face card (a jack, a queen or a king) and the second card is a king?

   (a) $\frac{3}{169}$ **(b)** $\frac{4}{13}$ **(c)** $\frac{4}{221}$ **(d)** $\frac{3}{221}$ **(e)** None of the above

7. A decorative coaster has a design of five circles that are tangent to each other, as shown. The largest circle has radius 2 inches. The two smaller circles each have radius 1 inch. Find the radius of the smallest (filled) circle.



(a) $1/4$

(b) $1/\pi$ **(c)** $10/3$ **(d)** $1/3$ **(e)** None of the above

8. What are the last three digits (the three least significant digits) in 2018^5?

(a) 368 **(b)** 568 **(c)** 768 **(d)** 968 **(e)** None of the above

9. How many positive 5 digit integers can be formed using only the digits 2, 0, 1 and 8 when in each number, each of the digits is used at least once?

(a) 120 **(b)** 180 **(c)** 200 **(d)** 240 **(e)** None of the above

10. What is the perimeter of a regular hexagon whose area is $18\sqrt{3}$ square units?

(a) 12 units **(b)** $12\sqrt{2}$ units **(c)** $12\sqrt{3}$ units

(d) $(6\sqrt{3}+4)$ units **(e)** None of the above

11. Suppose

$$1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\cdots=2018$$

and

$$1+\frac{1}{y}+\frac{1}{y^2}+\frac{1}{y^3}+\cdots=2019.$$

What is y/x?

(a) $\dfrac{2019}{2018}$ **(b)** $1-\dfrac{1}{2018^2}$ **(c)** $\dfrac{2017}{2019}$ **(d)** $1-\dfrac{1}{2019^2}$

(e) $\dfrac{2018\cdot 2019}{2017\cdot 2020}$

12. Let n be a positive integer such that

$$\frac{n^3 + 6n^2 + 25n + 391}{n + 4}$$

is an integer. How many possible values of n are there?

(a) 0 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** There are infinitely many possibilities for n

13. When multiplied out,

$$13! = 622\_020800$$

What is the missing digit?

(a) 3 **(b)** 5 **(c)** 7 **(d)** 9 **(e)** None of the above

14. Which of the following does *not* have a horizontal asymptote of $y = -1$?

(a) $y = e^{-3x} - 1$ **(b)** $y = \dfrac{3 - \ln x}{2 + \ln x}$ **(c)** $y = \log_2 x - 1$ **(d)** $y = \dfrac{1}{x} - 1$

(e) $y = 3^{-x} - 1$

15. Which of the following is equal to $\cos\left(\frac{\pi}{12}\right)$?

(a) $\dfrac{\sqrt{2}}{4}$ **(b)** $\dfrac{\sqrt{3} + \sqrt{2}}{4}$ **(c)** $\dfrac{\sqrt{6} + \sqrt{2}}{4}$ **(d)** $\dfrac{\sqrt{6} - \sqrt{2}}{4}$ **(e)** None of the above

16. Which of the following is the largest?

(a) $\cos\dfrac{\pi}{6}$ **(b)** $\log_2 1$ **(c)** $\log_2 5$ **(d)** $\tan\dfrac{\pi}{4}$ **(e)** $\sqrt{2}$

17. How many ways are there to arange five A's and fourteen B's if each A must be immediately followed by a B?

(a) $\binom{19}{5} + \binom{19}{4}$ **(b)** $\binom{19}{5}$ **(c)** $\binom{14}{5} \cdot \binom{14}{9}$

(d) $\binom{14}{5}$ **(e)** None of the above

18. A circle, an equilateral triangle and a square each have perimeter 12π. Which of the following give the three shapes in ascending order by area?

(a) △, ○, □ **(b)** ○, △, □ **(c)** □, △, ○ **(d)** △, □, ○ **(e)** ○, □, △

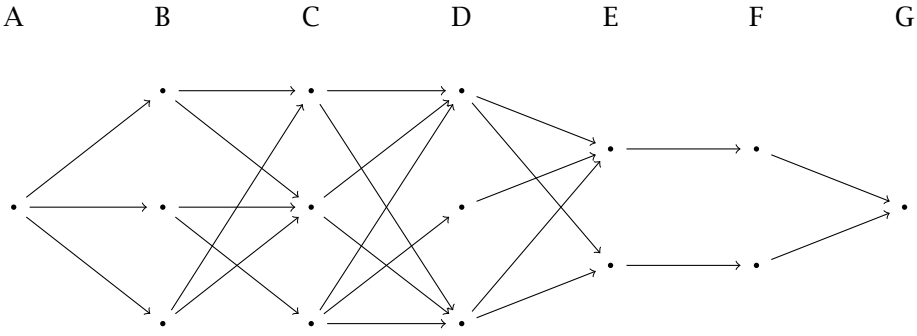
19. A car has wheels with radii 40cm. How many revolutions per minute must a wheel turn so that the car travels 50km/h?

(a) $\frac{6520}{\pi}$ **(b)** $\frac{3125}{3\pi}$ **(c)** $\frac{6520}{3}$ **(d)** $\frac{3125}{3}$ **(e)** None of the above

20. The point (x, y) lies on a circle with radius 3 and center at the origin. Find the maximal value of $x^2 + 3y^2 + 4x$.

(a) 22 **(b)** 24 **(c)** 36 **(d)** 27 **(e)** 29

21. In the diagram, A, B, ..., G refer to successive states through which a traveler must pass in order to get from A to G, moving from left to right. A path consists of a sequence of line segments leading from one state to the next. A path must always move to the next state until reaching state G. Determine the number of possible paths from A to G.



(a) 20 **(b)** 23 **(c)** 24 **(d)** 25 **(e)** 30

22. How many 10-digit strings of zeros and ones are there that do not contain any consecutive zeros?

    (a) 144 **(b)** 512 **(c)** 513 **(d)** 1280 **(e)** None of the above

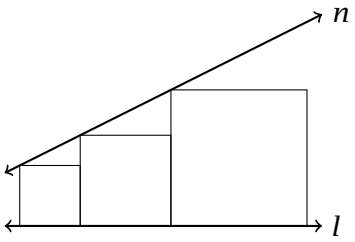
23. Find the value of $\sin(2\theta)$ if $\sin\theta + \cos\theta = 0.8$.

    (a) $-.36$ **(b)** $-.16$ **(c)** 0 **(d)** .16 **(e)** .36

24. The numbers x and y satisfy $2^x = 15$ and $15^y = 32$. What is the value of xy?

    (a) 3 **(b)** 4 **(c)** 5 **(d)** 6 **(e)** None of the above

25. Three adjacent squares with increasing side lengths sit on line l as shown, with line n passing through their top left corners. If the two smaller squares have side lengths of 4 and 6, what is the side length of the largest square?



    (a) 8

    (b) $\frac{26}{3}$ **(c)** 9 **(d)** 10 **(e)** None of the above