Saginaw Valley State University

2018 Math Olympics — Level I

1. What is the sum of the value(s) of k for which the graph of $y = x^2 + (k-3)x + k$ has exactly one x-intercept?

   (a) 10 **(b)** 9 **(c)** 0 **(d)** 3 **(e)** None of the above

2. Which of the following is the value of $\frac{2}{3} + \frac{1}{4} + \frac{2}{15} + \frac{1}{12} + \frac{2}{35} + \frac{1}{24} + \frac{2}{63} + \cdots + \frac{2}{(2017)(2019)}$?

   (a) $1 - \frac{1}{(2018)(2019)}$ **(b)** $\frac{3}{2} - \frac{4037}{(2018)(2019)}$ **(c)** $1 + \frac{1}{(2018)(2019)}$

   (d) $\frac{2018}{2019}$ **(e)** None of the above

3. Function f is defined in the following way:

   $$f(1) = 1$$
   $$f(k) = f(k-1) + k \text{ for an integer } k > 1$$

   What is $f(4000)$?

   (a) 16,004,000 **(b)** 8,002,000 **(c)** 80,000

   (d) 7999 **(e)** None of the above

4. Which of the following polynomials is a factor of the polynomial $x^3 + 3x^2y + 3xy^2 + y^3 - 8$?

   (a) $x^2 + 2xy + 2x + 2y + y^2 + 4$ **(b)** $(x + y - 2)^2$

   (c) $(x + 3y)$ **(d)** $(x + y)^3$

   (e) None of the above

5. How many positive divisors does $2^2 \cdot 3^3 \cdot 4^4 \cdot 5^5 \cdot 6^6 \cdot 7^7$ have?

   (a) 5040 **(b)** 20160 **(c)** 8160 **(d)** 7200 **(e)** None of the above

6. A two digit number is written at random. What is the probability that the sum of the digits is 5?

   (a) $\frac{5}{89}$ **(b)** $\frac{1}{18}$ **(c)** $\frac{6}{89}$ **(d)** $\frac{1}{15}$ **(e)** $\frac{4}{89}$

7. Whole numbers from 1 to 2018 are written in a row. How many times does the digit 8 appear?

   (a) 201 **(b)** 601 **(c)** 602 **(d)** 918 **(e)** None of the above

8. Find the least common multiple of 375, 175, 168 and 308.

   (a) 11550 **(b)** 231000 **(c)** 1155000

   (d) 3395700000 **(e)** None of the above

9. How many of the following equations have exactly one integer solution (and possibly any number of non-integer solutions)?

   ▷ $\sqrt{3x} = 6$

   ▷ $\sqrt{5x} = 9$

   ▷ $\sqrt{x} = x$

   ▷ $\sqrt{x} = 3x$

   (a) None **(b)** 1 **(c)** 2 **(d)** 3 **(e)** All 4

10. A large rectangle is subdivided into nine smaller rectangles as shown in the schematic drawing (not to scale). Five of the smaller rectangles have their perimeters written inside them. What is the perimeter of the large rectangle?

    (a) 18 **(b)** 34 **(c)** 42 **(d)** 64 **(e)** None of the above

	21	
9	15	12
	7	

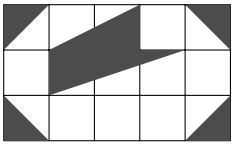
11. Each of 75 students in a science class is a member of at least one of the three clubs: Math club, Chemistry club and Physics club. Exactly 52 of them are in the Math club, exactly 32 of them are in the Chemistry club, and exactly 16 of them are in the Physics club. Exactly 15 of the students are in the Chemistry club only, and exactly 5 of them are in both the Math club and the Physics club, but not the Chemistry club. How many students are in all three of the clubs?

    (a) 3 **(b)** 5 **(c)** 9 **(d)** 15 **(e)** None of the above

12. How many positive 5 digit integers can be formed when using only the digits 2, 3, 6 and 8 so that in each number, each of the digits is used at least once?

    (a) 50 **(b)** 125 **(c)** 200 **(d)** 240 **(e)** None of the above

13. A rectangle is subdivided into 15 congruent squares as shown. Each square measures 4 inches on each side. How many square inches are in the unshaded area ?



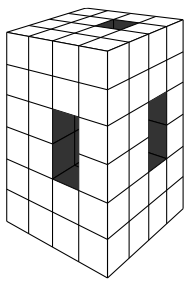
    (a) 7.5 **(b)** 120 **(c)** 10.5 **(d)** 168 **(e)** None of the above

14. If this pattern repeats indefinitely, what is the 2018th letter in the pattern?

    SAGINAWSAGINAWSAGINAWSAGINAWSAGINAWSAGINAW...

    (a) S **(b)** G **(c)** N **(d)** W **(e)** None of the above

15. The figure on the right consists of small cubes. How many of the these cubes are there? Assume that, if there is a "hole" on any surface, then that hole goes all the way through the figure, and that there are no other holes or cavities than the ones shown.



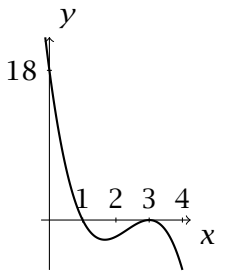
    (a) 74 **(b)** 78 **(c)** 89 **(d)** 91 **(e)** None of the above

16. An operation $\star$ on real numbers is associative, and $1 \star a = a \star 1 = a$ for any real number a. However, the operation is not commutative, in other words, in general, $a \star b \neq b \star a$. We also know that $7 \star 7 \star 7 = 1$, $5 \star 5 = 1$, and $5 \star 7 = 7 \star 7 \star 5$. Which of the following must be true about $5 \star 7 \star 7$?

(a) $5 \star 7 \star 7 = 7 \star 5$ **(b)** $5 \star 7 \star 7 = 7 \star 7 \star 5$ **(c)** $5 \star 7 \star 7 = 1$

(d) $5 \star 7 \star 7 = 7$ **(e)** None of the above

17. Which of the following expressions is a factored form of the third degree polynomial function $f(x)$ whose graph is given?



(a) $18(x-1)(x-3)^2$

(b) $-2(x-1)(x-3)^2$ **(c)** $18(x-1)^2(x-3)$

(d) $-6(x-1)^2(x-3)$ **(e)** None of the above

18. If all Martians are green aliens, and some two-headed creatures are Martians, which of the following statements must be true?

I. Some Martians are not two-headed creatures.

II. All Martians are two-headed creatures.

III. Some green aliens are two-headed creatures.

(a) I only **(b)** II only

(c) III only **(d)** I and III only

(e) None of them must be true

19. Which of these numbers is largest?

(a) $\sqrt[3]{3 \cdot \sqrt{7}}$ **(b)** $\sqrt[3]{7 \cdot \sqrt{3}}$ **(c)** $\sqrt{\sqrt[3]{3 \cdot 7}}$ **(d)** $\sqrt{3 \cdot \sqrt[3]{7}}$ **(e)** $\sqrt{7 \cdot \sqrt[3]{3}}$

20. What is the sum of the solutions of $x^2 - 5x = 14$?

(a) -5 **(b)** 5 **(c)** 9 **(d)** 19 **(e)** None of the above

21. A simplified form of the complex fraction $\dfrac{\frac{1}{x}+1}{-\frac{1}{x}+1}$ is

(a) $\dfrac{x+1}{1-x}$ **(b)** $\dfrac{x+1}{x-1}$ **(c)** $\dfrac{x-1}{1-x}$ **(d)** $\dfrac{x-1}{x-1}$ **(e)** None of the above

22. Which of the following equations is an equation of a circle with no intercepts and with center that is on the ray that bisects the first quadrant?

(a) $x^2-6x+y^2-6y=-14$ **(b)** $x^2-4x+y^2-4y=-4$

(c) $x^2+4x+y^2+4y=-9$ **(d)** $x^2-6x+y^2-4y=-12$

(e) None of the above

23. What are the last three digits (the three least significant digits) in 2018^5?

(a) 368 **(b)** 568 **(c)** 768 **(d)** 968 **(e)** None of the above

24. What is i^{2018}?

(a) 1 **(b)** i **(c)** -1 **(d)** $-i$ **(e)** None of the above

25. In December, a retailer raised the price of certain product by 25%. In January, the product went on sale so that the discounted price was the same as the original price of the product. How many percent was the discount?

(a) 20% **(b)** 22.5% **(c)** 25% **(d)** 27.5%

(e) It is impossible to determine without knowing the original price.