

Saginaw Valley State University  
2017 Math Olympics — Level I

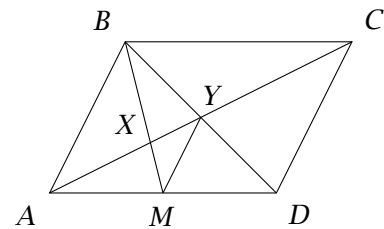
1. What is the last (least significant) digit of  $2^{2017}$ ?

- (a) 0    (b) 2    (c) 4    (d) 6    (e) 8

2. How many integers between 1 and 2017 are divisible by 5 but not 10?

- (a) 200    (b) 201    (c) 202    (d) 403    (e) 404

3. Given parallelogram  $ABCD$  (as shown) with area of  $ABCD = 24$  and diagonals  $\overline{AC}$  and  $\overline{BD}$  intersecting at point  $Y$ . The point  $M$  lies on  $AD$  so that the area of  $\triangle ABM$  is equal to the area of  $\triangle MBD$ . What is the area of  $MYCD$ ?



- (a) 6  
(b) 8    (c) 9    (d) 12    (e) None of the above

4. Find

$$\sum_{k=1}^{2017} i^k$$

where  $i^2 = -1$ .

- (a)  $-1$     (b)  $i$     (c)  $1$     (d)  $-i$     (e)  $1 + i$

5. Johnny, Dee Dee, Joey, Tommy, and Marky are in a band. The sums of the ages of each group of four of them are 132, 138, 113, 131, and 126. What is the age of the oldest of the band members?

- (a) 39    (b) 43    (c) 45    (d) 47    (e) 49

6. Determine the number of positive divisors of 18,800 that are divisible by 235.

- (a) 8    (b) 10    (c) 12    (d) 14    (e) 22

7. The sum of the radii of two circles is 10 cm. The circumference of the larger circle is 3 cm greater than the circumference of the smaller circle. Determine the difference between the area of the larger circle and the area of the smaller circle.
- (a)  $\pi$  cm<sup>2</sup>      (b)  $\frac{2\pi}{3}$  cm<sup>2</sup>      (c) 10 cm<sup>2</sup>      (d) 15 cm<sup>2</sup>      (e) 20 cm<sup>2</sup>
8. The sides  $a$ ,  $b$ , and  $c$  of a triangle satisfy  $\sqrt{a} + \sqrt{b} = \sqrt{c}$ . Which of the following describes the triangle?
- (a) acute                      (b) scalene                      (c) isosceles
- (d) equilateral              (e) None of the above
9. If the operation  $*$  is defined for all positive real numbers  $x$  and  $y$  by  $x * y = \frac{x+y}{xy}$ , which of the following must be true for positive  $x$ ,  $y$ , and  $z$ ?
- I.  $x * x = \frac{2}{x}$
- II.  $x * y = y * x$
- III.  $x * (y * z) = (x * y) * z$
- (a) I only                      (b) I and II only              (c) I and III only              (d) II and III only
- (e) all three
10. Two people who work full time and one who works half time are to work together on a project, but their total time allotted to the project is to be the equivalent to one and a half full time work days. If one of the full time workers is budgeted to give half of his work day to the project, and the other is budgeted to give one third of his work day, what part of the half-time worker's day should be given to the project?
- (a)  $\frac{1}{6}$       (b)  $\frac{1}{3}$       (c)  $\frac{9}{5}$       (d)  $\frac{4}{3}$       (e) None of the above
11. If four out of five numbers have a sum of 349 and the average of the five numbers is 70, find the fifth number.
- (a) 1              (b) -1              (c) -337              (d) 337              (e) None of the above
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12. Elmer drives from Springfield to Lafayette in three hours. Bugs takes a route that is five miles shorter and gets there in half the time. If  $r$  is the average rate for Elmer, which of the following is the average rate for Bugs in terms of  $r$ ?
- (a)  $(6r - 10)/3$       (b)  $(3r - 5)/6$       (c)  $(r - 5)/2$   
(d)  $3r/10$       (e) None of the above
13. If  $a \neq b$  and  $1/x + 1/a = 1/b$  then  $x =$
- (a)  $b - a$       (b)  $1/b - 1/a$       (c)  $1/a - 1/b$       (d)  $ab/(a - b)$   
(e) None of the above
14. If  $3x^2 - 4x + 5 = 0$  then  $(x - 2/3)^2 =$
- (a)  $-11/9$       (b)  $-1/3$       (c)  $19/9$       (d)  $25/9$       (e) None of the above
15. How many intersection points does the graph of  $y = x^2$  have with the circle centered at  $(0, 3)$  with radius 2.
- (a) none      (b) 1      (c) 2      (d) 3      (e) 4
16.  $2^{7/6} - 2^{2/3} =$
- (a)  $\sqrt[3]{4}(\sqrt{2} + 1)$       (b)  $\sqrt{2}$       (c)  $\sqrt{8}(\sqrt{2} - 1)$   
(d)  $\frac{\sqrt[3]{4}}{\sqrt{2}+1}$       (e) None of the above
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17. A collection of centipedes (100 legs), spiders (8 legs) and worms (no legs) has 224 legs and 28 heads, how many worms are there, if there is at least one of each?

- (a) 25    (b) 23    (c) 24    (d) 26    (e) None of the above

18. Abby, Benny, and Louie together have 100 pennies. If Abby had six times as many as she has now and Benny had one third as many as he has now, there would still be 100 pennies. Louie says, "It's not fair, I have fewer than 30." How many pennies does Louie have?

- (a) 15    (b) 25    (c) 28    (d) 29    (e) None of the above

19. Find the sum of the integers in the arithmetic sequence

$$11, 28, 45, 62, \dots, 2017$$

- (a) 119,652    (b) 119,663    (c) 120,655    (d) 120,666    (e) None of the above

20. If the lines  $ax + by = 7$  and  $ax - by = 9$  intersect at  $(1, 2)$ , what are  $a$  and  $b$ , respectively?

- (a) 9 and 7    (b)  $\frac{3}{2}$  and  $\frac{1}{2}$     (c) 7 and 9    (d) 8 and  $-\frac{1}{2}$     (e) None of the above

21. If  $AB$  is a number with digits (in base 10)  $A$  and  $B$ , such that when we place a decimal point between the digits we obtain one-third of the original number minus 14, what is  $AB$ ?

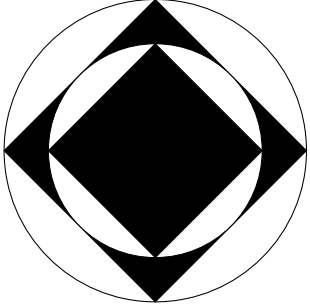
- (a) 11    (b) 53    (c) 60    (d) 120    (e) None of the above

22. In a certain class, 60% of students have competed in Math Olympics and 35% of students participated in the Physics Marathon. Exactly 10% of students did both. What percentage have done neither?

- (a) 5%    (b) 10%    (c) 15%    (d) 20%

(e) 0%, how could anyone possibly not participate in at least one of these cool events?

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23. When you divide a number by 210, you get 11. What do you get when you divide the same number by 14?
- (a) 11      (b) 66      (c) 154      (d) 165      (e) None of the above
24. Math Olympics has decided to branch into the snack industry. It sells two products, Pythagorean Peanut Mix, containing 35% granola by weight, and Gauss Granola Mix, containing 85% granola by weight. A customer wants a special mixture that contains 65% granola by weight. If they want 10 pounds of the mixture, how many pounds of each product is required?
- (a) 5 pounds of each
- (b) 4 pounds of Pythagorean Peanut Mix and 6 pounds of Gauss Granola Mix
- (c) 6 pounds of Pythagorean Peanut Mix and 4 pounds of Gauss Granola Mix
- (d) 2 pounds of Pythagorean Peanut Mix and 8 pounds of Gauss Granola Mix
- (e) None of the above
25. Kenny and Meghan found a cool dart board at a garage sale. It was a white circle in which there was inscribed a black square, in which there was an inscribed white circle, in which there was inscribed a black square, as shown. They were having a debate whether there was more black than white on the dartboard. Meghan said there was more white but Kenny was sure the black area was much larger. He thought that the black area was at least 1.5 times larger than the white area. Rich, who was also shopping at the same garage sale, thought that the black area was larger than the white area, but less than 1.5 times larger. The seller claimed the two areas were equal. Which one of them was right?
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- (a) Meghan was right, the white area is larger.
- (b) The seller was right, the two areas are equal.
- (c) Rich was right, the black area is larger, but less than 1.5 times larger.
- (d) Kenny was right, the black area is at least 1.5 times larger than the white area.
- (e) It is impossible to determine without knowing the diameter of the large circle.
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