SAGINAW VALLEY STATE UNIVERSITY
2015 MATH OLYMPICS LEVEL I

1. The value of $\frac{6}{\left(\sqrt{\sqrt{9}-\sqrt{3}}\right)^2}$ is

(a) 1 **(b)** $3-\sqrt{3}$ **(c)** $3+\sqrt{3}$ **(d)** $\frac{6}{3+\sqrt{3}}$ **(e)** $6(3+\sqrt{3})$

2. Suppose that $f(n) = 2f(n+1) - f(n-1)$ for all integers n and $f(1) = 4$ and $f(-1) = 2$. Evaluate $f(2)$.

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

3. Which of the following equations describes the set of all points (x, y) such that the product of the distance from (x, y) to $(-2, 0)$ and the distance from (x, y) to $(2, 0)$ is 4?

(a) $(x^2 + y^2)^2 = 32$ **(b)** $x^4 + 2x^2y^2 + y^4 + 8y^2 - 8x^2 + 12 = 0$
(c) $x^4 - 8x^2 + y^4 = 0$ **(d)** $(x^2 + y^2)^2 + 8(y^2 - x^2) = 0$
(e) none of the above

4. The average of three positive integers is 28. When two additional integers, s and t, are included, the average of all five integers is 34. What is the average of s and t?

(a) 90 **(b)** 80 **(c)** 70 **(d)** 60 **(e)** none of the above

5. What is the largest prime divisor of $2^{17} - 32$?

(a) 11 **(b)** 13 **(c)** 19 **(d)** 23 **(e)** 29

6. What is the smallest positive integer p for which $\sqrt{2^3 \times 5 \times p}$ is an integer?

(a) 1 **(b)** 2 **(c)** 5 **(d)** 10 **(e)** 20

7. 25 friends are going to order pizza. 7 of the friends like pepperoni on their pizza, 4 of those 7 like both pepperoni and mushrooms, while 3 of those 4 like olives as well. A total of 8 of the friends like mushrooms, and of those 8, 2 of them like mushrooms and olives but no pepperoni. Only 1 person likes only olives. How many of the friends like none of olives, pepperoni, and mushrooms on their pizza?

(a) 13 **(b)** 7 **(c)** 2 **(d)** None of them **(e)** none of the above

8. A bullet is fired at a target at a speed of 100 m/sec. 9 seconds later the sound of the bullet hitting the target comes back to the person firing the shot. If we take the speed of sound to be approximately 350 m/sec, how far away is the target?

(a) 100 meters **(b)** 200 meters **(c)** 700 meters
(d) 900 meters **(e)** none of the above

9. If $f(x) = \frac{x^2+1}{x^2-1}$, then $f\left(\frac{1}{x}\right)$ is equal to:

(a) $f(x)$ **(b)** $-f(x)$ **(c)** $-\frac{1}{f(x)}$ **(d)** 1 **(e)** none of the above

10. A standard playing card deck has 52 cards made up of 4 different suits with 13 kinds of cards (numbered 1 or "ace" through 10 and three face cards) in each suit. What is the minimum number of cards that must be drawn from a randomly shuffled deck in order to be sure of getting three cards of the same suit?

(a) 9 **(b)** 13 **(c)** 27 **(d)** 42 **(e)** none of the above

11. Maureen runs a catering service where she gives parties for a flat rate of \$600 for 50 guests or less. For any 5 guests over 50 the rate drops by 50 cents per person. What number of guests will maximize her revenue?

(a) 600 **(b)** 50 **(c)** 60 **(d)** 85 **(e)** none of the above

12. A chemist has three bottles, each containing a mixture of acid and water: bottle A contains 40 g of which 10% is acid, bottle B contains 50 g of which 20% is acid, and bottle C contains 50 g of which 30% is acid. She uses some of the mixture from each of the bottles to create a mixture with mass 60 g of which 25% is acid. Then she mixes the remaining contents of the bottles to create a new mixture. What percentage of the new mixture is acid?

(a) 20% **(b)** 17.5% **(c)** 15.5% **(d)** 25%
(e) none of the above

13. Four years ago, Diane was three times as old as Jean was. In five years, Diane will be twice as old as Jean will be. What is the sum of the ages of Diane and Jean?

(a) 25 **(b)** 32 **(c)** 42 **(d)** 44 **(e)** 54

14. Seven women and seven men attend a party. At this party, each man shakes hands with each other person once. Each woman shakes hands only with men. How many handshakes took place at the party?
(a) 49 **(b)** 70 **(c)** 91 **(d)** 133 **(e)** 182

15. Given that $p(x) = (x-1)(x^3+2x-3)+(x-5)(x^3+2x-3)$, what is the sum of the real solutions of $p(x) = 0$?
(a) 4 **(b)** 6 **(c)** -2 **(d)** 7
(e) $p(x) = 0$ has no solution

16. Given that (x, y) satisfies $x^2 + y^2 = 9$, what is the largest possible value of $x^2 + 3y^2 + 4x$?
(a) 22 **(b)** 24 **(c)** 36 **(d)** 27 **(e)** 29

17. Mr. and Mrs. Alpha, Mr. and Mrs. Beta, and Mr. and Mrs. Gamma are standing in a line. How many ways are there line them up so that none of them are standing next to their spouse?
(a) 144 **(b)** 240 **(c)** 288 **(d)** 432 **(e)** none of the above

18. The sides of a right triangle are $a, 2a + 2d$ and $2a + 3d$, with a and d both positive. The ratio of a to d is:
(a) 5:1 **(b)** 27:2 **(c)** 4:1 **(d)** 1:5 **(e)** none of the above

19. Let f be a function defined on positive integers that satisfies $f(k+2) = 4f(k)$. Which of the following could be an equation of f?
(a) $f(k) = 4^k$ **(b)** $f(k) = 4k$ **(c)** $f(k) = \frac{3}{4}(2)^k + \frac{1}{4}(-2)^k$
(d) $f(k) = \frac{1}{2}(4)^k - \frac{1}{2}(-4)^k$ **(e)** none of the above

20. The midpoints of the sides of a triangle are $(1, 1), (4, 3)$, and $(3, 5)$. Find the area of the triangle.
(a) 14 **(b)** 16 **(c)** 18 **(d)** 20 **(e)** 22

21. The side lengths of an equilateral triangle and a square are integers. If the triangle and the square have the same perimeter, which of the following is a possible side length of the triangle?
(a) 1 **(b)** 10 **(c)** 18 **(d)** 20 **(e)** 25

22. If $f(x) = ax^2 + bx + c$, and if the y-intercept of f is 1 and the x-intercepts of f are 2 and 3, then $a + b + c =$

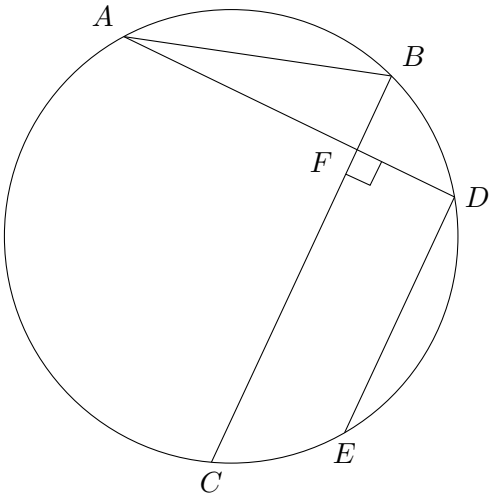
(a) 2 **(b)** 6 **(c)** $\frac{1}{3}$ **(d)** $\frac{5}{3}$ **(e)** none of the above

23. In the diagram, the rectangle is divided into nine smaller rectangles. The areas of five of these rectangles are given. Determine the area of the rectangle labelled R.

3	1	
	2	R
5		10

(a) 30 **(b)** 20 **(c)** 15 **(d)** 14 **(e)** 12

24. In the diagram, AB and BC are chords of the circle with $AB < BC$. If D is the point on the circle such that AD is perpendicular to BC and E is the point on the circle such that DE is parallel to BC, what is $\angle EAC + \angle ABC =$?



(a) 60° **(b)** 75° **(c)** 90° **(d)** 105° **(e)** 120°

25. For how many integers n, with $2 \leq n \leq 80$, is $\frac{(n-1)(n)(n+1)}{8}$ equal to an integer?

(a) 10 **(b)** 20 **(c)** 59 **(d)** 39 **(e)** 49