**SAGINAW VALLEY STATE UNIVERSITY**
**2014 MATH OLYMPICS LEVEL II**

**1.** Define a function $f$ by $f(0) = c$, $f(-101) = -25348$ and $f(n-1) = f(n) + 5n - 1$ for any integer $n$ ( for example, $f(-1) = f(0) - 1$). The value of $c =$?

**(a)** 6 **(b)** 5 **(c)** 4 **(d)** 3 **(e)** 0

**2.** Let $f(x, y)$ be defined by $f(x, 0) = x$ and $f(x, y+1) = f(f(x, y), y)$: which of the following is the largest?

**(a)** $f(10, 15)$ **(b)** $f(11, 14)$ **(c)**$f(12, 13)$ **(d)** $f(13, 12)$ **(e)** $f(14, 11)$

**3.** How many integers can be expressed as a sum of three distinct numbers chosen from the set $\{4, 7, 10, 13, \ldots, 46\}$?

**(a)** 36 **(b)** 37 **(c)** 42 **(d)** 43 **(e)** 45

**4.** How many solutions does the equation

$$\sin(x)\sin(2x)\sin(3x)\cdots\sin(11x)\sin(12x) = 0$$

have in the interval $(0, \pi]$?

**(a)** 12 **(b)** 24 **(c)** 46 **(d)** 68 **(e)** none of the above

**5.** Six tickets numbered 1 through 6 are placed in a box. Two tickets are randomly selected and removed together. What is the probability that the smaller of the two numbers on the tickets selected is less than or equal to 4?

**(a)** $\frac{14}{15}$ **(b)** $\frac{13}{15}$ **(c)** $\frac{4}{5}$ **(d)** $\frac{3}{5}$ **(e)** none of the above

**6.** It is known that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. What is the value of $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$?

**(a)** $\frac{\pi^2}{36}$ **(b)** $\frac{\pi^2}{12}$ **(c)** $\frac{\pi^2}{8}$ **(d)** $\frac{\pi^2}{7}$ **(e)** none of the above

**7.** If $f(x)$ satisfies $2f(x) + f(1-x) = x^2$ for all $x$, then $f(x) =$

**(a)** $\frac{x^2-3x+1}{2}$ **(b)** $\frac{x^2+8x-3}{9}$ **(c)** $\frac{x^2+2x-1}{3}$ **(d)** $\frac{4x^2+3x-2}{6}$
**(e)** none of the above

**8.** If Sam and Peter are among 6 men who are seated at random in a row, the probability that exactly 2 men are seated between them is

**(a)** 1/10 **(b)** 1/8 **(c)** 1/5 **(d)** 1/4 **(e)** none of the above

**9.** If $\frac{1}{\cos x} - \tan x = 3$, what is value of $\sin x$?

**(a)** $-4/5$ **(b)** $4/5$ **(c)** 1 **(d)** 0 **(e)** none of the above

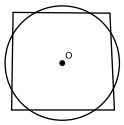
**10.** Define $f(x) = \sin^6 x + \cos^6 x + k(\sin^4 x + \cos^4 x)$ for some real number $k$. Then the value of $k$ for which $f(x)$ is constant for all values of $x$ is

**(a)** 0 **(b)** $\frac{3}{2}$ **(c)** $\frac{-3}{2}$ **(d)** $\frac{5}{2}$ **(e)** $\frac{7}{2}$

**11.** The function $f(x)$ satisfies the equation $f(x) = f(x-1) + f(x+1)$ for all values of $x$. If $f(1) = 1$ and $f(2) = 3$, what is the value of $f(2014)$?

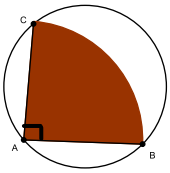
**(a)** 3 **(b)** 1 **(c)** $-1$ **(d)** $-3$ **(e)** none of the above

**12.** In the diagram, the circle and the square have the same center $O$ and equal areas. The circle has radius 1 and intersects one side of the square at $P$ and $Q$. What is the length of $PQ$?



**(a)** $\sqrt{4-\pi}$ **(b)** 1 **(c)** $\sqrt{2}$ **(d)** $2-\sqrt{\pi}$ **(e)** $4-\sqrt{\pi}$

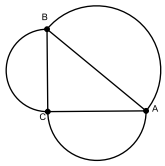
**13.** In the figure, $ABC$ is a quarter of a circular pizza with center $A$ and radius 20 cm. The piece of pizza is placed on a circular pan with $A, B$ and $C$ touching the circumference of the pan, as shown. What fraction of the pan is covered by the piece of pizza?




**(a)** $1/4$ **(b)** $1/\sqrt{2}$ **(c)** $1/2$ **(d)** $2\sqrt{2}$ **(e)** none of the above

**14.** A right triangle $ABC$ is given. Semicircles are constructed with the sides of the triangle as diameters, as shown below. Suppose the area of the largest semicircle is 36 and the area of the smallest one is 16. What is the area of the other one?

**(a)** 20 **(b)** 24 **(c)** 25 **(d)** 26 **(e)** none of the above




**15.** How many positive integers less than 1000 have only odd digits?

**(a)** 125 **(b)** 155 **(c)** 165 **(d)** 150 **(e)** none of the above

**16.** The tips of a five-pointed star are to be painted red, white and blue. How many ways can this be done if no adjacent points can be the same color?

**(a)** 30 **(b)** 10 **(c)** 40 **(d)** 45 **(e)** none of the above

**17.** If $x > 0$, $x \neq 1$, $a = \log_2(x)$ and $b = \log_x(7)$, what is $\log_{14}(x)$?
**(a)** $ab$ **(b)** $a + b$ **(c)** $b - a$ **(d)** $a/b$ **(e)** none of the above

**18.** Suppose that $a, b, c$, and $d$ are positive integers that satisfy the equations

$$ac + bd = 34$$

$$ad + bc = 43$$

What is the value of $a + b + c + d$?
**(a)** 19 **(b)** 18 **(c)** 17 **(d)** 16 **(e)** none of the above

**19.** What is the coefficient of $x^3$ in the polynomial $(1 + x + x^2)^{20}$?
**(a)** 20 **(b)** 380 **(c)** 1520 **(d)** 2340 **(e)** none of the above

**20.** Suppose $f(x) = ax + b$ and $a$ and $b$ are real numbers. We define $f_1(x) = f(x)$ and $f_{n+1}(x) = f(f_n(x))$ for all positive integers $n$. If $f_7(x) = 128x + 381$, what is the value of $a + b$?
**(a)** 7 **(b)** 5 **(c)** 3 **(d)** 2 **(e)** 1

**21.** Let $a, b$ and $c$ be real numbers which satisfy the three equations below.

$$a + \frac{1}{bc} = \frac{1}{5}, \qquad b + \frac{1}{ac} = \frac{-1}{15}, \qquad c + \frac{1}{ab} = \frac{1}{3}.$$

What is the value of the quotient $\frac{c-b}{c-a}$?
**(a)** $-5$ **(b)** $-3$ **(c)** 1 **(d)** 3 **(e)** 5

**22.** Given are two parallel lines of distance 1 apart and a circle of radius 2. The circle is tangent to one of the lines and cuts the other line. The area of the circular cap between the two parallel lines is $a\frac{\pi}{3} - b\sqrt{3}$. Find the sum $a + b$ of the two integers $a$ and $b$.
**(a)** 3 **(b)** 4 **(c)** 5 **(d)** 6 **(e)** 7

**23.** The average of three consecutive multiples of 3 is $a$.
The average of four consecutive multiples of 4 is $a + 27$.
The average of the smallest and largest of these seven integers is 42.
The value of $a$ is
**(a)** 15 **(b)** 18 **(c)** 24 **(d)** 27 **(e)** none of the above

**24.** Let $AB$ and $CD$ be two chords of a circle that intersect at a point $P$. Suppose that $AP = 4, PB = 6, CP = 2, PD = 12$ and $\angle APC = 90°$. What is the radius of the circle?
**(a)** $4\sqrt{3}$ **(b)** $3\sqrt{6}$ **(c)** 8 **(d)** $5\sqrt{2}$ **(e)** none of the above

**25.** For how many integers $n$ is the value of $\sqrt{\frac{n}{50-n}}$ is an integer?
**(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5