SAGINAW VALLEY STATE UNIVERSITY
2014 MATH OLYMPICS LEVEL I

1. If $2^x = 2(16^{12}) + 2(8^{16})$, what is the value of x?
(a) 50 **(b)** 37 **(c)** 28 **(d)** 25 **(e)** none of the above

2. Suppose the operation $*$ is defined on the set of integers by $a * b = 1 - a - b$. Then for every two integers a and b, the value of $a * (a * b)$ is the same as
(a) 0 **(b)** 2 **(c)** a **(d)** b **(e)** none of the above

3. Determine all values of k, with $k \neq 0$, for which the parabola $y = kx^2 + (5k + 3)x + (6k + 5)$ has its vertex on the x-axis.
(a) -3, -3 **(b)** -2, -6 **(c)** -1, -9 **(d)** 0, -12 **(e)** 1, 3

4. Nana and Boris took the Math Olympics Contest. If we double Nana's score we get 60 more than Boris's score. If we double Boris's score we get 90 more than Nana's score. Determine the average of their two scores.
(a) 90 **(b)** 80 **(c)** 70 **(d)** 60 **(e)** none of the above

5. The units digit in the product $(5^2 + 1)(5^3 + 1)(5^{23} + 1)$ is
(a) 3 **(b)** 5 **(c)** 6 **(d)** 2 **(e)** 1

6. The number of integers x for which the value of $\frac{-6}{x+1}$ is an integer is
(a) 2 **(b)** 4 **(c)** 6 **(d)** 8 **(e)** 9

7. On Monday, 10% of the students at SVSU were absent and 90% were present. On Tuesday, 10% of those who were absent on Monday were present and the rest of those absent on Monday were still absent. Also, 10% of those who were present on Monday were absent and the rest of those present on Monday were still present. What percentage of the students at SVSU were present on Tuesday?
(a) 99% **(b)** 91% **(c)** 90% **(d)** 88% **(e)** 82%

8. Let n be the smallest positive integer whose digits have a product of 2000. The sum of the digits of n is

(a) 21 **(b)** 23 **(c)** 25 **(d)** 27 **(e)** 29

9. Find the sum of all values of x that satisfy $|x+1|+3|x-2|+5|x-4|=20$.

(a) 2 **(b)** 5 **(c)** 6 **(d)** 9 **(e)** 11

10. You have 10 coins, all of different weights and you can weigh them only in pairs in a two-pan balance. What is the minimal numbers of weighings needed to find the heaviest coin?

(a) 45 **(b)** 12 **(c)** 10 **(d)** 9 **(e)** 5

11. In his last will, a farmer asked that his horses be distributed among his four sons. The oldest was to get one third of the herd, the second oldest, one fourth of the herd, and each of the two youngest ones was to get one fifth of the herd. When the sons read the will, they were puzzled because none of them were going to get an integer number of horses. At that moment, they discovered that a baby horse had just been born. Each son would receive an integer number of horses, but the baby horse would be left over. How many horses did the farmer have originally?

(a) 29 **(b)** 59 **(c)** 89 **(d)**119 **(e)** 239

12. An athlete covers three consecutive miles by swimming the first, running the second and cycling the third. He runs twice as fast as he swims and cycles one and a half times as fast as he runs. He takes ten minutes longer than he would if he had cycled the whole three miles. How many minutes does he take?

(a) 16 minutes **(b)** 22 minutes **(c)** 30 minutes
(d) 46 minutes **(e)** none of the above

13. Given that a and b are positive real numbers with $a+b=4$, what is the minimum value of $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)$?

(a) 2 **(b)** 3 **(c)** 4 **(d)** $\frac{8}{3}$ **(e)** $\frac{9}{4}$

14. You own eleven pairs of socks, all different, and all of the socks are individually jumbled in a drawer. One morning you rummage through the drawer and continue to pull out socks until you have a matching pair. How many socks must you pull out to guarantee having a matching pair?

(a) 3 **(b)** 6 **(c)** 11 **(d)** 12 **(e)** none of the above

15. Given that $f(x) = (x^5 - 1)(x^3 + 1), g(x) = (x^2 - 1)(x^2 - x + 1)$, and $h(x)$ is a polynomial such that $f(x) = g(x)h(x)$, what is the value of $h(1)$?

(a) 0 **(b)** 2 **(c)** 3 **(d)** 5 **(e)** undefined

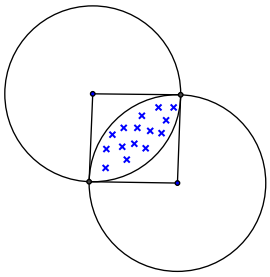
16. The set of all x such that $(|x| - 2)(1 + x) > 0$ is exactly

(a) $|x| > 2$ **(b)** $-2 < x < 1$ **(c)** $x > 2$ **(d)** $-2 < x < -1$
(e) none of the above

17. A doll that is 4 inches tall represents a 64 inch person. A doll house is to be made using the same scaling factor that is an exact model of a real house. If a rectangular rug in the real house has an area of $32ft^2$, what size will the rug be in the doll house?

(a) $18in^2$ **(b)** $24in^2$ **(c)** $512in^2$ **(d)** $42.66in^2$ **(e)** $2in^2$

18. What is the area of the region common to two unit circles with centers that are $\sqrt{2}$ apart?



(a) $\frac{\pi}{2} - 1$ **(b)** $\frac{1}{2}(1 - \frac{\pi}{4})$ **(c)** $1 - \frac{\pi}{4}$ **(d)** $\frac{\pi}{2}$ **(e)** none of the above

19. Suppose that $f(x) = ax + b$ where a and b are real numbers. Given that $f(f(f(x))) = 8x + 21$, what is the value of $a + b$?

(a) 6 **(b)** 5 **(c)** 4 **(d)** 3 **(e)** none of the above

20. Curt's car gets 3 more miles per gallon during highway driving than it does during city driving. On a recent trip, Curt drove 136 miles on the highway and 155 miles in the city, using a total of 9 gallons of gasoline. How many miles per gallon does Curt's car get during city driving?

(a) 30 **(b)** 31 **(c)** 32 **(d)** 34 **(e)** 35

21. The area bounded by two concentric circles is 5π square centimeters. The difference between the radii of the circles is 1 centimeter. What is the radius of the smaller circle, in centimeters?

(a) 6 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4

22. Point A on a coordinate plane has coordinates $(2, 0)$. Point B lies in the first quadrant on the line $x - y = 1$. If the distance from point A to point B is 5 units, what is the sum of the coordinates of point B?

(a) $\sqrt{41}$ **(b)** 9 **(c)** 3 **(d)** 11 **(e)** none of the above

23. In the diagram, the perimeter of the semicircular region is 20. (The perimeter includes both the semicircular arc and the diameter.) The area of the region is

(a) $\frac{1}{2}\pi\left(\frac{20}{\pi+1}\right)^2$ **(b)** $\frac{1}{2}\pi\left(\frac{10}{\pi+1}\right)^2$ **(c)** $200(\pi + 4)$ **(d)** $400(\pi + 4)$
(e) none of the above

24. The area of a circle circumscribed about a regular hexagon is 200π. What is the area of the hexagon?

(a) $300\sqrt{3}$ **(b)** $600\sqrt{3}$ **(c)** $60\sqrt{3}$ **(d)** 600 **(e)** 1200

25. For how many integers n is the value of $\frac{n}{50-n}$ the square of an integer?

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5