SAGINAW VALLEY STATE UNIVERSITY
2012 MATH OLYMPICS LEVEL I

1. Which of the following polynomials is a factor of $x^4 + x^2y^2 + y^4$?
(a) $x^2(x^2 + y^2)$ **(b)** $x^2 - xy + y^2$ **(c)** $x^2 + y^2$
(d) The polynomial is prime (or is not factorable) **(e)** none of the above

2. $19,320 = (2 \times 69)(14 \times 10)$. Which of the following numbers is a factor of $19,321$?
(a) 141 **(b)** 137 **(c)** 139 **(d)** $19,321$ is prime
(e) none of the above

3. Suppose that $(r_1, 0)$ and $(r_2, 0)$ are the x-intercepts of a parabola. If the equation of the parabola has a leading coefficient of 1, what is the y-intercept of the parabola.
(a) $(0, r_1 + r_2)$ **(b)** $(0, -r_1 - r_2)$ **(c)** $(0, r_1 r_2)$ **(d)** $(0, -r_1 r_2)$
(e) none of the above

4. Daphne starts walking north at a constant rate of 4 mph. Edmund leaves the same point at the same time and walks east at a constant rate of 3 mph. How long will it take for the distance between them to be 1 mile?
(a) 5 minutes **(b)** 12 minutes **(c)** $\frac{1}{7}$th of an hour
(d) 7 minutes **(e)** none of the above

5. $\frac{\frac{1}{5}-1}{\frac{1+\sqrt{5}}{5}}$ is equal to which of the following numbers?
(a) $\sqrt{5} - 1$ **(b)** $1 + \sqrt{5}$ **(c)** 1 **(d)** $1 - \sqrt{5}$
(e) none of the above

6. What are the slopes of the two tangent lines from the origin in the plane to the circle of radius $1/2$ centered at $(1, 1)$?
(a) $\frac{1 \pm \sqrt{2}}{2}$. **(b)** $1 \pm \sqrt{2}$ **(c)** $\frac{4 \pm \sqrt{7}}{3}$. **(d)** $2 \pm \sqrt{2}$
(e) none of the above

7. The door to the computer room at a school has a keycode. The combination is a sequence of 5 numbers. A student forgot his code. However, he did remember five clues. These are what those clues were:

(a) The fifth number plus the third number equals fourteen.
(b) The fourth number is one more than the second number.
(c) The first number is one less than twice the second number.
(d) The second number plus the third number equals ten.
(e) The sum of all five numbers is 30.

What is the fifth number?

(a) 8 **(b)** 4 **(c)** 7 **(d)** 5 **(e)** none of the above

8. Three boxes are presented to you. One contains \$1000, the other two are empty. Each box has a clue written on it as to its contents and only one message is telling the truth, the other two are lying. If the first box says, "The money is not here", the second box says "The money is in the first box" and the third box says, "The money is not here", which box has the money?

(a) the first box **(b)** the second box **(c)** the third box
(d) more than one box could contain the money **(e)** none of the boxes

9. In the following "equation" each letter represents a digit (between 0 and 9). Different letters represent different digits and S is not 0. Determine the digit represented by each of the used letters so that the addition is correct.

$$\begin{array}{r} STORE \\ +STORE \\ +STORE \\ \hline \overline{TEASE} \end{array}$$

(a) $E = 3, O = 5, R = 7, S = 2, T = 8$ **(b)** $E = 4, O = 3, R = 7, S = 2, T = 8$
(c) $E = 0, O = 2, R = 7, S = 1, T = 3$ **(d)** $E = 5, O = 3, R = 7, S = 2, T = 8$
(e) none of the above

10. Suppose you have 30 coins which are nickels, dimes, or quarters. The total amount of money is \$4.15. One day you find a magic wand that has the ability to change nickels into dimes and dimes into quarters. After you wave the wand over the money, you find you have \$6.00. How many dimes did you have originally?

(a) 8 **(b)** 9 **(c)** 7 **(d)** 10 **(e)** none of the above

11. You purchase 6 new tires for your four wheel car. Each tire is designed to provide a maximum of 40,000 miles of use. Assume you can rotate tires on and off the vehicle at any time. What is the maximum mileage you could get from these tires?

(a) 40,000 miles **(b)** 50,000 miles **(c)** 60,000 miles
(d) 80,000 miles **(e)** none of the above

12. The faces of a solid figure are all triangles. The figure has 11 vertices. At each of six vertices, four faces meet and at each of the other five vertices, six faces meet. How many faces does the figure have?

(a) 20 **(b)** 17 **(c)** 16 **(d)** 19 **(e)** none of the above

13. It takes a horse and a goat two hours to eat 20 pounds of hay. If it takes the horse three more hours than the goat to eat 20 pounds of hay, how long does it take the horse to eat the 20 pounds of hay?

(a) 5 hours **(b)** 6 hours **(c)** 1 hour **(d)** 4 hours and 20 minutes
(e) none of the above

14. A club of 150 members is holding an arm wrestling contest. When a member loses, the member is out of the contest. There are no ties. How many games must be played to determine the arm wrestling champion?

(a) 149 **(b)** 150 **(c)** 148 **(d)** 75 **(e)** none of the above

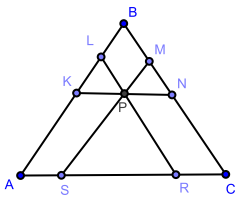
15. For all x such that $x \neq 3, \frac{7}{3}, \frac{-7}{3}, 0$, what is $\frac{5}{x-3} - \frac{80}{3x^2-2x-21}$ equal to?

(a) $\frac{15}{3x-7}$ **(b)** $\frac{45x-105}{49-9x^2}$ **(c)** $\frac{\frac{15}{x}}{3+\frac{7}{x}}$ **(d)** all of the above
(e) none of the above

16. Let x, y be real numbers with $x + y = 1$ and $(x^2 + y^2)(x^3 + y^3) = 12$. What is the value of $x^2 + y^2$?

(a) $\sqrt{2}$ **(b)** $\sqrt{3}$ **(c)** 2 **(d)** 3 **(e)** none of the above

17. P is a point inside the triangle $\triangle ABC$. Lines are drawn through P parallel to the sides of the triangle. The areas of the three resulting triangles $\triangle PMN, \triangle PLK$ and $\triangle PRS$ are 9, 25 and 81, respectively. What is the area of $\triangle ABC$?
Note that the figure below is not drawn to the scale.



(a) 181 **(b)** 203 **(c)** 289 **(d)** 250 **(e)** none of the above

18. Nine scientists are working on a secret project. They wish to lock up the documents in a cabinet so that the cabinet can be opened when and only when *five or more of the scientists are present*. For this purpose a certain number of locks are installed on the cabinet and each of the scientists is given keys to some of these locks. Each key can open exactly one lock. Thus, for the cabinet to be opened (i) any five of the scientists have to be present and (ii) the keys to all of the locks on the cabinet have to be among the set of all keys given to the present five scientists. What is the smallest number of locks needed?

(a) 126 **(b)** 63 **(c)** 142 **(d)** 280 **(e)** none of the above

19. Suppose that $f(x) = x^x$ and $g(x) = x^{2x}$. Which of the functions below is equal to $f(g(x))$?

(a) $x^{x^{2x}}$ **(b)** $x^{2x^{2x+1}}$ **(c)** $x^{2x^{2x}}$ **(d)** x^{4x^3} **(e)** none of the above

20. Fred, reporting for the school paper, knows that Arthur, Bing and Claude were the top three finalists in the talent show, but he doesn't know who won first place, who was second or who was third place. He does know three things: 1) If Arthur did not win first place, then Bing did. 2) If Bing did not win second place, then Arthur came in last. 3) If Claude came in last, then Arthur did not come in first. Which of the following is the correct order.

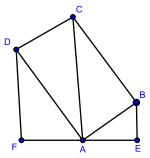
(a) First: Arthur; Second: Bing; Third: Claude
(b) First: Arthur; Second: Claude; Third: Bing
(c) First: Bing; Second: Arthur; Third: Claude
(d) First: Bing; Second: Claude; Third: Arthur
(e) Cannot be determined from the given information

21. Suppose the least common multiple of two numbers, x and 3675 is equal to 121275, and their greatest common factor is equal to 75. Which range of values below contains x?

(a) $(0, 1000)$ **(b)** $(1000, 2000)$ **(c)** $(3000, 4000)$ **(d)** $(2000, 3000)$

(e) none of the above

22. In the figure below, $ABCD$ is a rectangle. The points F, A, and E lie on a straight line. The segments DF, BE, and CA are all perpendicular to FE. The length of DF is 15 and the length of BE is 6. What is the length of FE?



(a) $6\sqrt{10}$ **(b)** $3\sqrt{10}$ **(c)** $3\sqrt{5}$ **(d)** $3\sqrt{2}$ **(e)** $\sqrt{10}$

23. The medians AN and BM of the triangle $\triangle ABC$ intersect at the point G. The vertex C and the points M, G, N are on a circle. The length of $AB = a$. Find the length of the third median CP.

(a) $\frac{a\sqrt{2}}{2}$ **(b)** $\frac{a\sqrt{3}}{2}$ **(c)** $a\sqrt{2}$ **(d)** $\frac{a}{2}$ **(e)** none of the above

24. Let $\triangle ABC$ be a right triangle ($\measuredangle C = 90°$). The perpendicular bisector l through the midpoint O of the hypothenuse AB intersects one of the legs at a point K and the extension of the other leg at a point D. If $\mid OK \mid = m$, and $\mid OD \mid = n$, find $\mid AB \mid$.

(a) $2\sqrt{mn}$ **(b)** $\sqrt{mn}$ **(c)** $2\sqrt{\frac{m}{n}}$ **(d)** $2\sqrt{\frac{n}{m}}$ **(e)** none of the above

25. A problem to remember the year 2011: let $x = .01234567891011\ldots998999$ where the digits are obtained by listing the numbers 0-999 in order. What is the 2011^{th} digit to the right of the decimal place?

(a) 7 **(b)** 6 **(c)** 5 **(d)** 4 **(e)** none of the above