SOLUTIONS OF 2012 MATH OLYMPICS LEVEL I

1. Which of the following polynomials is a factor of $x^4 + x^2y^2 + y^4$?

(a) $x^2(x^2 + y^2)$ **(b)** $x^2 - xy + y^2$ **(c)** $x^2 + y^2$

(d) The polynomial is prime (or is not factorable) **(e)** none of the above

The answer is: **(b)**

Solution:

Method 1: $x^4+x^2y^2+y^4 = x^4+x^2y^2+x^2y^2-x^2y^2+y^4 = x^4+2x^2y^2+y^4-x^2y^2 = (x^2+y^2)^2-(xy)^2 = (x^2+y^2-xy)(x^2+y^2+xy)$.

Method 2: One can also factor $x^4+x^2y^2+y^4$ by factoring x^6-y^6 in two different ways. First factoring as a difference of squares and then factoring the two remaining factors as a sum and difference of cubes we get:
$x^6-y^6 = (x^3)^2-(y^3)^2 = (x^3-y^3)(x^3+y^3) = (x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$.
Factoring as a difference of cubes first we get:
$x^6-y^6 = (x^2)^3-(y^2)^3 = (x^2-y^2)(x^4+x^2y^2+y^4) = (x-y)(x+y)(x^4+x^2y^2+y^4)$.
Equating the two factorizations and dividing both sides by $(x-y)(x+y)$ shows that $(x^4+x^2y^2+y^4) = (x^2+xy+y^2)(x^2-xy+y^2)$.

2. $19,320 = (2\times 69)(14\times 10)$. Which of the following numbers is a factor of $19,321$?

(a) 141 **(b)** 137 **(c)** 139 **(d)** 19,321 is prime

(e) none of the above

The answer is: **(c)**

Solution: Solution: $19320 = 138 \times 140 = (139-1)(139+1) = 139^2 - 1$.
So $19320 + 1 = 139^2$.

3. Suppose that $(r_1, 0)$ and $(r_2, 0)$ are the x-intercepts of a parabola. If the equation of the parabola has a leading coefficient of 1, what is the y-intercept of the parabola.

(a) $(0, r_1 + r_2)$ **(b)** $(0, -r_1 - r_2)$ **(c)** $(0, r_1 r_2)$ **(d)** $(0, -r_1 r_2)$

(e) none of the above

The answer is: **(c)**

Solution: Since r_1 and r_2 are the roots and the leading coefficient is 1, the equation of the parabola is $y = (x - r_1)(x - r_2)$. So when $x = 0, y = r_1 r_2$.

4. Daphne starts walking north at a constant rate of 4 mph. Edmund leaves the same point at the same time and walks east at a constant rate of 3 mph. How long will it take for the distance between them to be 1 mile?

(a) 5 minutes **(b)** 12 minutes **(c)** $\frac{1}{7}$th of an hour

(d) 7 minutes **(e)** none of the above

The answer is: **(b)**

Solution: Let D be the distance Daphne has walked, E the distance Edmund has walked, d the distance between them, and t the time they have been walking in hours.

Using distance = rate × time, $D = 4t$ and $E = 3t$. Using the Pythagorean theorem, $d = \sqrt{D^2 + E^2} = \sqrt{(4t)^2 + (3t)^2} = \sqrt{16t^2 + 9t^2} = \sqrt{25t^2} = 5t$.

So $d = 1$ mile when $t = \frac{1}{5}$hours$= \frac{60 \text{ minutes}}{1 \text{ hour}} = 12$ minutes.

5. $\frac{\frac{1}{5}-1}{\frac{1+\sqrt{5}}{5}}$ is equal to which of the following numbers?

(a) $\sqrt{5} - 1$ **(b)** $1 + \sqrt{5}$ **(c)** 1 **(d)** $1 - \sqrt{5}$

(e) none of the above

The answer is: **(d)**

Solution:

$$\frac{\frac{1}{5} - 1}{\frac{1+\sqrt{5}}{5}} = \frac{(\frac{1}{5} - 1) \cdot 5}{(\frac{1+\sqrt{5}}{5}) \cdot 5} = \frac{1 - 5}{1 + \sqrt{5}} = \frac{-4(1 - \sqrt{5})}{(1 + \sqrt{5})(1 - \sqrt{5})} = \frac{-4(1 - \sqrt{5})}{1 - 5} = 1 - \sqrt{5}.$$

6. What are the slopes of the two tangent lines from the origin in the plane to the circle of radius 1/2 centered at $(1, 1)$?

(a) $\frac{1\pm\sqrt{2}}{2}$. **(b)** $1 \pm \sqrt{2}$ **(c)** $\frac{4\pm\sqrt{7}}{3}$. **(d)** $2 \pm \sqrt{2}$

(e) none of the above

The answer is: **(c)**

Solution: The equation for the circle of radius one half centered at $(1, 1)$ is

$$(x - 1)^2 + (y - 1)^2 = 1/4.$$

The tangent lines are the values of a for which $y = ax$ meets the circle in exactly one point. Substituting in $y = ax$ into the equation of the circle gives $(1 + a^2)x^2 - 2(1 + a)x + 2 = 1/4$, one wants exactly one real solution which means that the radical $\sqrt{4(1 + a)^2 - 4(1 + a^2)(7/4)} = \sqrt{-3a^2 + 8a - 3}$ appearing in the quadratic formula must be 0. Solving this new quadratic equation gives

$$a = \frac{4 \pm \sqrt{7}}{3}.$$

7. The door to the computer room at a school has a keycode. The combination is a sequence of 5 numbers. A student forgot his code. However, he did remember five clues. These are what those clues were:

(a) The fifth number plus the third number equals fourteen.
(b) The fourth number is one more than the second number.
(c) The first number is one less than twice the second number.
(d) The second number plus the third number equals ten.
(e) The sum of all five numbers is 30.
What is the fifth number?

(a) 8 **(b)** 4 **(c)** 7 **(d)** 5 **(e)** none of the above

The answer is: **(a)**

Solution: We can set up a system of equations for x_i = the i-th number in the code, i.e., x_1 is the first number, x_2 is the second number, etc. The system will then be:

$x_3 + x_5 = 14$
$-x_2 + x_4 = 1$
$-x_1 + 2x_2 = 1$
$x_2 + x_3 = 10$
$x_1 + x_2 + x_3 + x_4 + x_5 = 30.$

If we add the third equation to the last one we obtain

$$3x_2 + x_3 + x_4 + x_5 = 31,$$

while adding together the first, the second and the third equations gives

$$2x_3 + x_4 + x_5 = 25.$$

If we then subtract the last two equations we obtain an equation for x2 and x3

$$3x_2 - x_3 = 6.$$

The latter together with the forth equation of the initial system give

$$x_2 + x_3 = 10$$

$$3x_2 - x_3 = 6,$$

which when added show that $4x_2 = 16$. Hence, $x_2 = 4$, $x_3 = 6$ and after a few substitutions we find that the numbers in order are 7, 4, 6, 5, 8.

8. Three boxes are presented to you. One contains $1000, the other two are empty. Each box has a clue written on it as to its contents and only one message is telling the truth, the other two are lying. If the first box says, "The money is not here", the second box says "The money is in the first box" and the third box says, "The money is not here", which box has the money?

(a) the first box **(b)** the second box **(c)** the third box
(d) more than one box could contain the money **(e)** none of the boxes

The answer is: **(c)**

Solution: If the money is in the first box, then the messages on both the second and third boxes are true, which contradicts the fact that only one message is telling the truth. If the money is in the second box, then the messages on the first and third box are true, which is again a contradiction. Hence, the money must be in the third box. In that case, the message on box one is true, while the messages on boxes two and three are false as required.

9. In the following "equation" each letter represents a digit (between 0 and 9). Different letters represent different digits and S is not 0. Determine the digit represented by each of the used letters so that the addition is correct.

$$\begin{array}{r} STORE \\ +STORE \\ +STORE \\ \hline TEASE \end{array}$$

(a) $E = 3, O = 5, R = 7, S = 2, T = 8$ **(b)** $E = 4, O = 3, R = 7, S = 2, T = 8$
(c) $E = 0, O = 2, R = 7, S = 1, T = 3$ **(d)** $E = 5, O = 3, R = 7, S = 2, T = 8$

(e) none of the above

The answer is: **(d)**

Solution: We shall use repeatedly that three times a number between 0 and 9 is a number between 0 and 27. Adding the units it follows $3 \times E$ equals E, or $10 + E$ or $20 + E$. Solving the three equations we find that either $E = 0$, or $E = 5$, or $E = 10$, which leaves only the first two possibilities. Now we look at the possibilities for S. Since $3 \times S$ has to be less than 10 it follows that S is either 1, 2 or 3. Consider first the case, $S = 1$. The possible values of T are 3, 4 or 5. If $T = 3$ then, taking into account the value of S, it follows that $E = 9$, which is a contradiction. If $T = 4$ then the possible values of E obtained from $3 \times T$ plus any carry over are 2, 3 or 4. This is a contradiction. Finally, if $T = 5$, then E has to be 5, 6 or 7. This is again not possible since different letters represent different digits, which gives a contradiction in the first case, while the last two possibilities are excluded from the already determined values of E. The case $S = 3$ leads to a contradiction as well since then $T = 9$ which leads to a contradiction since $3 \times T = 3 \times 9 = 27$. Consider now the case, $S = 2$. The possible values of T are 6, 7 or 8. The first two possibilities are excluded by the possible values of E and the value of S. The last choice gives the solution

$$\begin{array}{r} 28375 \\ +28375 \\ 28375 \\ \hline 85125 \end{array}$$

10. Suppose you have 30 coins which are nickels, dimes, or quarters. The total amount of money is \$4.15. One day you find a magic wand that has the ability to change nickels into dimes and dimes into quarters. After you wave the wand over the money, you find you have \$6.00. How many dimes did you have originally?

(a) 8 **(b)** 9 **(c)** 7 **(d)** 10 **(e)** none of the above

The answer is: **(b)**

Solution: Let x, y and z be the number of nickels, dimes and quarters respectively. Then $x + y + z = 30$ since the total of coins is 30. If the total amount of money is \$4.15, we have $.05x + .1y + .25z = 4.15$, i.e., $5x + 10y + 25z = 415$. If the nickels are changed into dimes and dimes into quarters, we have now x dimes and $y + z$ quarters and this yields the equation $10x + 25y + 25z = 600$. Solving the system of equations

$$x + y + z = 30$$

$$5x + 10y + 25z = 415$$

$$10x + 25y + 25z = 600$$

gives $x = 10$, $y = 9$ and $z = 11$.

11. You purchase 6 new tires for your four wheel car. Each tire is designed to provide a maximum of 40,000 miles of use. Assume you can rotate tires on and off the vehicle at any time. What is the maximum mileage you could get from these tires?

(a) 40,000 miles **(b)** 50,000 miles **(c)** 60,000 miles
(d) 80,000 miles **(e)** none of the above

The answer is: **(c)**

Solution: The car uses 4 tires and to get the maximum mileage all the 6 tires must be used for a total of $6 \times 40,000 = 240,000$ miles. Now to get the maximum mileage from the tires, just divide the total by 4, i.e., 240,000/4=60,000 miles.

12. The faces of a solid figure are all triangles. The figure has 11 vertices. At each of six vertices, four faces meet and at each of the other five vertices, six faces meet. How many faces does the figure have?

(a) 20 **(b)** 17 **(c)** 16 **(d)** 19 **(e)** none of the above

The answer is: **(e)**

Solution: Each face is counted as part of three vertices, so the number of faces f equals $f = (6 \times 4 + 5 \times 6)/3 = 54/3 = 18$. Alternatively, we can use Eulers formula $v - e + f = 2$ where v is the number of vertices, e is the number of edges and f is the number of faces. Here $v = 11$. We need to first determine e then we can find f. If four faces meet at a vertex this means that there are four edges at this vertex. Similarly if six faces meet at a vertex, there are six edges at that vertex. Since each edge starts and ends at a different vertex, then $2e = 6 \times 4 + 5 \times 6 = 54$. Hence $e = 27$ and $f = e - v + 2 = 27 - 11 + 2 = 18$.

13. It takes a horse and a goat two hours to eat 20 pounds of hay. If it takes the horse three more hours than the goat to eat 20 pounds of hay, how long does it take the horse to eat the 20 pounds of hay?

(a) 5 hours **(b)** 6 hours **(c)** 1 hour **(d)** 4 hours and 20 minutes
(e) none of the above

The answer is: **(b)**

Solution: Let x be the amount of time it takes for the horse to eat the hay. Notice that $x > 3$ since it takes the horse three more hours than the goat to eat 20 lb of hay. Assuming that the horse and the goat are eating at a constant rate, then the horse eats at the rate of $20/x$ lb/hr, while the goat eats at the rate of $20/(x-3)$ lb/hr. We know that if both are eating together they will eat 20 lb of hay in 2 hours, i.e.,

$$2\left(\frac{20}{x} + \frac{20}{x-3}\right) = 20.$$

Thus

$$\frac{2x-3}{x^2-3x} = \frac{1}{2}.$$

Simplifying we find $x^2 - 3x = 4x - 6$, hence $x^2 - 7x + 6 = 0$. So either $x = 1$ or $x = 6$. Since $x > 3$, it must be $x = 6$.

14. A club of 150 members is holding an arm wrestling contest. When a member loses, the member is out of the contest. There are no ties. How many games must be played to determine the arm wrestling champion?
(a) 149 **(b)** 150 **(c)** 148 **(d)** 75 **(e)** none of the above

The answer is: **(a)**
Solution: Note that each game eliminates one player. Since there are no ties and 149 players are eliminated there are 149 games.

15. For all x such that $x \neq 3, \frac{7}{3}, \frac{-7}{3}, 0$, what is $\frac{5}{x-3} - \frac{80}{3x^2-2x-21}$ equal to?
(a) $\frac{15}{3x-7}$ **(b)** $\frac{45x-105}{49-9x^2}$ **(c)** $\frac{\frac{15}{x}}{3+\frac{7}{x}}$ **(d)** all of the above
(e) none of the above

The answer is: **(c)**
Solution:

$$\begin{aligned}\frac{5}{x-3} - \frac{80}{3x^2-2x-21} &= \frac{5}{x-3} - \frac{80}{(x-3)(3x+7)} \\ &= \frac{5(3x+7)-80}{(x-3)(3x+7)} \\ &= \frac{15x-45}{(x-3)(3x+7)}\end{aligned}$$

Now when $x \neq 3, \pm\frac{7}{3}, 0$, we do have

$$\frac{5}{x-3} - \frac{80}{3x^2-2x-21} = \frac{15(x-3)}{(x-3)(3x+7)} = \frac{15}{3x+7}$$

So it does not satisfy (a). The next line below shows that it also does not satisfy (b):

$$\frac{45x-105}{49-9x^2} = \frac{15(3x-7)}{(7+3x)(7-3x)} = \frac{-15}{3x+7}.$$

However,

$$\frac{\frac{15}{x}}{3+\frac{7}{x}} = \frac{\frac{15}{x}\cdot x}{(3+\frac{7}{x})\cdot x} = \frac{15}{3x+\frac{7x}{x}} = \frac{15}{3x+7}$$

Hence it does satisfy (c).

16. Let x, y be real numbers with $x+y=1$ and $(x^2+y^2)(x^3+y^3)=12$. What is the value of x^2+y^2?

(a) $\sqrt{2}$ **(b)** $\sqrt{3}$ **(c)** 2 **(d)** 3 **(e)** none of the above

The answer is: **(d)**

Solution 1: Let $u = x^2+y^2$. Then $u+2xy = x^2+y^2+2xy = (x+y)^2 = 1$ and $u(u-xy) = (x^2+y^2)(x^2+y^2-xy) = (x^2+y^2)(x^3+y^3)/(x+y) = 12$. Thus from $xy = \frac{1-u}{2}$, we have $u(u-\frac{1-u}{2}) = 12$. Hence after simplification, we just have to solve the quadratic equation $3u^2-u-24=0$. So $u=3$.

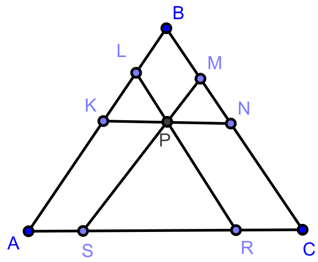
Solution 2: Since $x^2+y^2 = (x+y)^2-2xy$, $(x^2+y^2)(x^3+y^3) = ((x+y)^2-2xy)(x+y)((x+y)^2-3xy) = (1-2xy)(1-3xy) = 12$.

Let $xy = u$, then $(1-2u)(1-3u) = 6u^2-5u+1 = 12$. Solve for u, $u = \frac{11}{6}$ and $u=-1$.

Since x, y are real roots of $(t-x)(t-y) = t^2-(x+y)t+xy = t^2-t+u = 0$, the discriminant is $\sqrt{(}1-4u)$

$1-4u>0$ gives $u=-1$. Thus, $x^2+y^2 = (x+y)^2-2xy = 1+2 = 3$.

17. P is a point inside the triangle $\triangle ABC$. Lines are drawn through P parallel to the sides of the triangle. The areas of the three resulting triangles $\triangle PMN, \triangle PLK$ and $\triangle PRS$ are 9, 25 and 81, respectively. What is the area of $\triangle ABC$?
Note that the figure below is not drawn to the scale.



(a) 181 **(b)** 203 **(c)** 289 **(d)** 250 **(e)** none of the above

The answer is: **(c)**

Solution: $\triangle PMN, \triangle KLP$ and $\triangle SPR$ are similar triangles. We shall use that if we have two similar triangles with the ratio of linear elements (such as lengths of sides etc.) equal to λ, then the ratio of their areas is λ^2. Let $PN = x, PK = y$ and $SR = z$. From the similarity it follows, $y = 5/3x$ and $z = 3x$. Note that $\triangle ABC$ is also similar to $\triangle PMN$ and its base is $x + y + z$, so the similarity coefficient is $(x + y + z)/x = 17/3$. So the areas satisfy the relation $A(\triangle ABC) = (17/3)^2 A(\triangle PMN) = 17^2 = 289$.

18. Nine scientists are working on a secret project. They wish to lock up the documents in a cabinet so that the cabinet can be opened when and only when *five or more of the scientists are present.* For this purpose a certain number of locks are installed on the cabinet and each of the scientists is given keys to some of these locks. Each key can open exactly one lock. Thus, for the cabinet to be opened (i) any five of the scientists have to be present and (ii) the keys to all of the locks on the cabinet have to be among the set of all keys given to the present five scientists. What is the smallest number of locks needed?

(a) 126 **(b)** 63 **(c)** 142 **(d)** 280 **(e)** none of the above

The answer is: **(a)**

Solution: In the first step we will find a number of locks so that no less locks can solve the problem. Since no set of four scientists can open the cabinet for each combination of five scientists there must be at least one lock that cannot be opened by this group of four people. In this way to every combination of four scientists we associate a set of missing keys. Furthermore, since every five scientists can open the cabinet the missing keys of any two different combinations of four scientists have to be two disjoint sets. For otherwise there will be two groups of four scientists missing the same key, so there will be five scientists missing a key, which is a contradiction.

There are $\binom{9}{4} = 126$ combinations of four scientists, so there must be at least 126 locks on the cabinet. Next, we turn to the second step, where we shall show that 126 locks do solve the problem. For this we have to show that there is a way to distribute keys from 126 locks in a way which satisfies conditions (i) and (ii). It will be essential that

$$(*) \qquad \binom{9}{4} = \binom{9}{5},$$

which is true since selecting any five is the same as designating the remaining four. So, lets make five keys for each of the 126 locks. Then for each of the 126 combinations of five scientists (here we use $(*)$) give a key to the same lock to each of the scientists in the combination. In this way, each group of five scientists has a lock which they and only they can open. In particular, the remaining four cannot open it, so no less than five scientists can open the cabinet. We still have to show that every five scientists can open the cabinet, i.e., each group of five scientists has 126 different keys among them. For this we note that given any lock there is a distinguished combination of five scientists who all have keys to it. On the other hand, if we take any five scientists, at least one of them will belong to the distinguished combination since there are nine scientists overall. This proves the final claim.

19. Suppose that $f(x) = x^x$ and $g(x) = x^{2x}$. Which of the functions below is equal to $f(g(x))$?
(a) $x^{x^{2x}}$ **(b)** $x^{2x^{2x+1}}$ **(c)** $x^{2x^{2x}}$ **(d)** x^{4x^3} **(e)** none of the above

The answer is: **(b)**
Solution: $f(g(x)) = (x^{2x})^{x^{2x}} = x^{2xx^{2x}} = x^{2x^{1+2x}}$.

20. Fred, reporting for the school paper, knows that Arthur, Bing and Claude were the top three finalists in the talent show, but he doesn't know who won first place, who was second or who was third place. He does know three things: 1) If Arthur did not win first place, then Bing did. 2) If Bing did not win second place, then Arthur came in last. 3) If Claude came in last, then Arthur did not come in first. Which of the following is the correct order.
(a) First: Arthur; Second: Bing; Third: Claude
(b) First: Arthur; Second: Claude; Third: Bing
(c) First: Bing; Second: Arthur; Third: Claude
(d) First: Bing; Second: Claude; Third: Arthur
(e) Cannot be determined from the given information

The answer is: **(d)**

Solution: There are six permutation of three elements. From (1) we know that we have only the 4 choices given above where Claude is not in first place. From (2) we are down to (a) or (d). (3) rules out (a), so the only choice is (d).

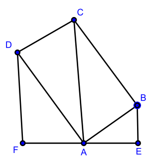
21. Suppose the least common multiple of two numbers, x and 3675 is equal to 121275, and their greatest common factor is equal to 75. Which range of values below contains x?

(a) $(0, 1000)$ **(b)** $(1000, 2000)$ **(c)** $(3000, 4000)$ **(d)** $(2000, 3000)$
(e) none of the above

The answer is: **(d)**

Solution: One can directly get the value of x with the fact that $3675x = 121,275 \times 75$. Hence $x = 2475$.

22. In the figure below, $ABCD$ is a rectangle. The points F, A, and E lie on a straight line. The segments DF, BE, and CA are all perpendicular to FE. The length of DF is 15 and the length of BE is 6. What is the length of FE?



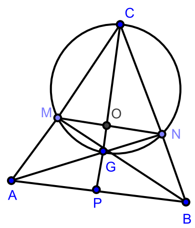
(a) $6\sqrt{10}$ **(b)** $3\sqrt{10}$ **(c)** $3\sqrt{5}$ **(d)** $3\sqrt{2}$ **(e)** $\sqrt{10}$

The answer is: **(a)**

Solution: Since $ABCD$ is a rectangle its two diagonals bisect each other in equal parts. In particular BD is split in half by AC. Since DF, CA and BE are parallel it follows that CA splits BD and FE in parts which have the same ratios. Therefore A is the midpoint between A and E. On the other hand, since the angle $\angle DAB = 90°$ and using again that DF, CA and BE are parallel it follows that the triangles $\triangle DFA$ and $\triangle AEB$ are similar. If we set $AF = x$ from the similarity of the triangles we obtain $x/15 = 6/x$, i.e., $x^2 = 90$ or $x = 3\sqrt{10}$. Thus, $FE = 2x = 6\sqrt{10}$.

23. The medians AN and BM of the triangle $\triangle ABC$ intersect at the point G. The vertex C and the points M, G, N are on a circle. The length of $AB = a$. Find the length of the third median CP.
(a) $\frac{a\sqrt{2}}{2}$ **(b)** $\frac{a\sqrt{3}}{2}$ **(c)** $a\sqrt{2}$ **(d)** $\frac{a}{2}$ **(e)** none of the above

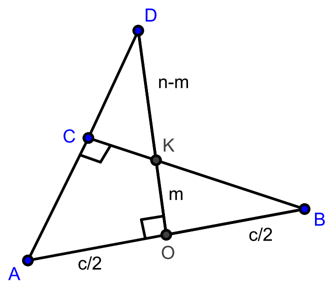
The answer is: **(b)**



Solution:
Let O be the point of intersection of MN and OG are two intersecting chords in the circle, we have $MO \cdot ON = CO \cdot OG$. But $MO = \frac{1}{2}AP = \frac{1}{2}(\frac{1}{2}a) = \frac{a}{4}$, $NO = \frac{1}{2}PB = \frac{1}{2}(\frac{1}{2}a) = \frac{a}{4}$, $CO = \frac{1}{2}CP$ and $OG = OP - GP = \frac{1}{2}CP - \frac{1}{3}CP = \frac{1}{6}CP$. Thus $\frac{a}{4}\frac{a}{4} = \frac{1}{2}CP\frac{1}{6}CP$, i.e., $\frac{a^2}{4} = \frac{1}{3}CP^2$. Hence $CP = \frac{a\sqrt{3}}{2}$.

24. Let $\triangle ABC$ be a right triangle ($\measuredangle C = 90°$). The perpendicular bisector l through the midpoint O of the hypothenuse AB intersects one of the legs at a point K and the extension of the other leg at a point D. If $\mid OK \mid= m$, and $\mid OD \mid= n$, find $\mid AB \mid$.
(a) $2\sqrt{mn}$ **(b)** $\sqrt{mn}$ **(c)** $2\sqrt{\frac{m}{n}}$ **(d)** $2\sqrt{\frac{n}{m}}$ **(e)** none of the above

The answer is: **(a)**
Solution:



Note that triangles $\triangle AOD$ and $\triangle KOB$ are similar. So $\frac{OK}{OA} = \frac{OB}{OD}$, i.e., $\frac{m}{c/2} = \frac{c/2}{n}$. Thus $c^2 = 4mn$ and $c = 2\sqrt{mn}$.

25. A problem to remember the year 2011: let $x = .01234567891011\ldots998999$ where the digits are obtained by listing the numbers 0-999 in order. What is the 2011^{th} digit to the right of the decimal place?

(a) 7 **(b)** 6 **(c)** 5 **(d)** 4 **(e)** none of the above

The answer is: **(b)**

Solution: There are 10 digits in the expansion among the one digit numbers and $90 \times 2 = 180$ digits among the two-digit numbers. Hence we are looking for the digit among the three digit numbers which is in the 1821^{th} place since 2011 - 190 = 1821. As $3 \times 607 = 1821$ and the 607^{th} three digit number is 607+99 = 706, the digit in the 2011^{th} place is 6.