SAGINAW VALLEY STATE UNIVERSITY
2011 MATH OLYMPICS-LEVEL I

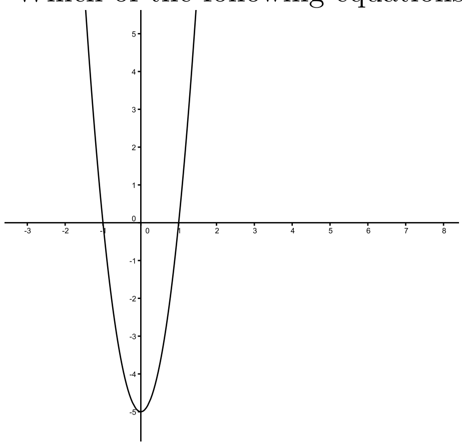
1. The Easter Bunny creates a giant chocolate Easter egg that weighs 20 pounds. It is the exact same shape as a chocolate egg weighing 5 ounces. The egg that weighs 5 ounces is wrapped in 9 in^2 of foil. How much foil needs to be used to wrap the giant egg?

(a) 36 ft^2 **(b)** 16 ft^2 **(c)** 4 ft^2 **(d)** 1 ft^2
(e) none of the above

2. Which of the following equations is the equation of a circle whose center is on the line that bisects the first quadrant and that has no intercepts?

(a) $x^2 - 6x + y^2 - 6y = -9$ **(b)** $x^2 + 6x + y^2 + 6y = -14$
(c) $x^2 - 6x + y^2 - 4y = -12$ **(d)** $4x^2 - 24x + 4y^2 - 24y = -47$
(e) none of the above

3. Which of the following equations could be the equation of the graph below.



(a) $y = -5x^2 - 5$ **(b)** $y = x^2 - 4x - 5$
(c) $y = x^4 + x^2 - 5$ **(d)** $y = -(x-1)(x+1)(x-5)$
(e) none of the above

4. How many triples of real numbers (x, y, z) satisfy the equations

$$xy = z, \qquad yz = x, \qquad zx = y?$$

(a) 5 **(b)** 1 **(c)** 4 **(d)** infinitely many **(e)** none of the above

5. Let a and b be real numbers satisfying the equations

$$a + b = 1, \qquad a^3 + b^3 = 2011.$$

What is $a^2 + b^2$?

(a) 1 **(b)** 2010 **(c)** 1341 **(d)** 1851 **(e)** none of the above

6. Which of the following quadratic polynomials f satisfies the property: "$f(n)$ is an integer whenever n is an integer"?

(a) $f(x) = x^2 + \frac{x}{2} + \frac{1}{2}$ **(b)** $f(x) = \frac{x^2}{2} + \frac{x}{2} + \frac{1}{2}$ **(c)** $f(x) = \frac{x^2}{2} + \frac{x}{2}$
(d) $f(x) = x^2 - \frac{x}{2} - \frac{1}{2}$ **(e)** none of the above

7. A drawer contains red socks and black socks. When two socks are drawn at random, the probability that both are red is $\frac{1}{2}$. How small can the number of socks in the drawer be if the number of black socks is even?

(a) 7 **(b)** 14 **(c)** 21 **(d)** 25 **(e)** none of the above

8. What is the smallest number of coins (pennies, nickels, dimes, quarters, and half dollars) with which you can pay out any amount from 1 cent to 99 cents?

(a) 8 **(b)** 9 **(c)** 10 **(d)** 12 **(e)** none of the above

9. A fair coin is tossed 10 times. What is the probability that exactly five of the tosses come up heads and five of them tails?

(a) $\frac{1}{2}$ **(b)** $\frac{63}{256}$ **(c)** $\frac{64}{256}$ **(d)** $\frac{5}{128}$ **(e)** none of the above

10. How many zeros are at the end of 17!? (Recall that $n!$ is the product of the first n positive integers, that is $2! = 2 \cdot 1$, $3! = 3 \cdot 2 \cdot 1$, and so on)

(a) 2 **(b)** 3 **(c)** 17 **(d)** 21 **(e)** none of the above

11. A train moving 45 miles per hour meets and is passed by a train moving 35 miles per hour in the opposite direction. A passenger in the first train sees that the second train takes 6 seconds to pass him. How long is the second train?

(a) 701 feet **(b)** 698 feet **(c)** 704 feet **(d)** 710 feet
(e) none of the above

12. Each morning Boris walks to school at a constant rate. At one-fourth of the way he passes the tractor station. At one-third of the way, the railroad station. At the tractor station its clock shows 7:30 A.M., and at the railroad station its clock shows 7:35 A.M. When does Boris reach school?

(a) 8:10 A.M. **(b)** 8:15 A.M. **(c)** 8:20 A.M. **(d)** 8:25 A.M.
(e) none of the above

13. If Peter Piper can pick a peck of pickled peppers in 2 hours, and Tom Thumb can do it in 3 hours, and Mary Mary can do it in 5 hours, how long will it take them working all together?

(a) 31/3 hours **(b)** 30/31 of an hour **(c)** 16/15 hours
(d) one hour and 2 minutes **(e)** none of the above

14. If r_1 and r_2 are solutions of the equation $x = a\sqrt{x + k^2}$, then $r_1 + r_2 =$?
(a) a^2 **(b)** $\frac{k^2}{1-a}$ **(c)** 0 **(d)** $a\sqrt{a^2 + 4ak^2}$ **(e)** none of the above

15. For all x such that $x \neq 5, \frac{3}{2}, \frac{-3}{2}, 0$, what is $\frac{2}{x-5} - \frac{14}{2x^2-13x+15}$ equal to?

(a) $\frac{4}{2x-3}$ **(b)** $\frac{8x+12}{4x^2-9}$ **(c)** $\frac{\frac{4}{x}}{2-\frac{3}{x}}$
(d) all of the above **(e)** none of the above

16. Let x, y, z be real numbers. Suppose that $(x + 1)(y + 1)(z + 1) \neq 0$, and

$$\frac{x}{x+1} + \frac{y}{y+1} + \frac{z}{z+1} = 1.$$

Find all possible values of the quantity $2xyz + xy + yz + zx$ for all x, y, z with the above properties.

(a) 1 **(b)** 0, 2, or 3 **(c)** 0 **(d)** infinitely many values
(e) none of the above

17. Suppose a glass has a square base of side 2 units and height 2 units. A spider is standing at the center of the bottom of the glass. The spider wants to reach a delicious ant that is standing at one of the top corners of the glass. Assume the spider walks at a constant speed on the surface of the glass and the ant, unaware of the danger, does not move. What distance should the spider walk to have her meal as quickly as possible?

(a) $2 + \sqrt{2}$ **(b)** $1 + \sqrt{5}$ **(c)** $\sqrt{10}$ **(d)** $2\sqrt{2}$ **(e)** none of the above

18. A quartet with 4 different instruments wants to play something (silence doesn't count) with each possible combination of instruments. How many ways can this be done?

(a) 15 **(b)** 10 **(c)** 24 **(d)** 18 **(e)** none of the above

19. Which x is a solution of the equation $x^3 + 3x - 4 = 0$?

(a) $x = (2+\sqrt{5})^{1/3} + (2-\sqrt{5})^{1/3}$ **(b)** $x = (2+\sqrt{5})^{1/3} - (2-\sqrt{5})^{1/3}$

(c) 0 **(d)** 2 **(e)** none of the above

20. Find real numbers A, B and C so that for all real numbers $x \neq 0, 3, -1$, the following identity holds

$$\frac{1}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1}$$

(a) $A = -1/3, B = 1/12, C = 1/14$ **(b)** $A = 1/3, B = -1/12, C = 1/14$

(c) $A = -1/2, B = 1/4, C = 1/12$ **(d)** $A = -1/3, B = 1/12, C = 1/4$

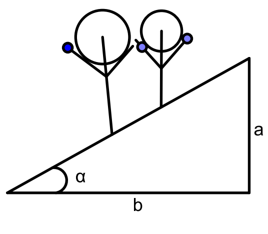
(e) none of the above

21. An equation of the parabola through the three points $(0,1)$, $(1,4)$, $(2,9)$ is:

(a) $f(x) = -x^2 + 4x + 1$ **(b)** $f(x) = x^2 - 2x + 1$ **(c)** $f(x) = 2x^2 + 2x + 1$

(d) $f(x) = x^2 + 2x + 1$ **(e)** none of the above

22. The unit used for measuring the steepness of a hill is the "grade". A grade of a to b means the hill rises a vertical units for every b horizontal units. If at some point, the hill is $3ft$ above the horizontal and the angle of elevation of α to that point is $30°$, what is the approximate grade to the nearest integer of this hill?

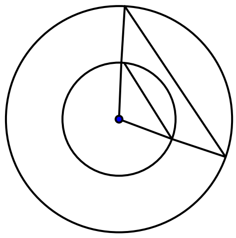


(a) 3 to 5 **(b)** 3 to 6 **(c)** 3 to 7 **(d)** 3 to 8 **(e)** 7 to 10

23. A circular corral was constructed with $600ft$ of barbed wire fence. Another circular corral with twice the area was constructed with the same type of fence. How much barbed wire was needed for the second?

(a) $1200ft$ **(b)** $600\sqrt{2}$ **(c)** 2400 **(d)** $1200\sqrt{2}$ **(e)** none of the above

24. Two concentric circles are drawn, the smaller with radius r and the larger with radius R. A central angle is drawn, such that the rays intersect each circle as shown in the figure. Given that the area of the triangle in the larger circle is three times the area of the corresponding triangle in the smaller circle, give the area lying between the two circles in terms of r.



(a) $2\pi r^2$ **(b)** $\sqrt{3}\pi r^2$ **(c)** $(\sqrt{3}-1)\pi r^2$ **(d)** πr^2 **(e)** none of the above

25. A problem to celebrate the new year 2011, what are the last three digits of 2011^5?

(a) 121 **(b)** 051 **(c)** 131 **(d)** 641 **(e)** none of the above