Saginaw Valley State University

2010 Math Olympics – Level I

1. For positive real numbers x, y, and z, which of the following is equivalent to $x^{-\frac{1}{2}}y^{\frac{2}{3}}z^{\frac{3}{5}}$?

   (a) $\sqrt[30]{(xyz)^{23}}$ **(b)** $x^2\sqrt[15]{y^{10}z^9}$ **(c)** $\sqrt[30]{x^{-1}y^2z^3}$

   (d) $\frac{\sqrt[30]{x^{15}y^{20}z^{18}}}{x}$ **(e)** None of the above

2. If f is a nonzero function (this means there is at least one x with $f(x) \neq 0$) of real numbers such that $f(x+y) = f(x)f(y)$, what are the possible values for $f(0)$?

   (a) any real number is possible **(b)** any positive real number is possible

   (c) $f(0)$ must be 0 **(d)** $f(0)$ must be 1

   (e) $f(0)$ could be 0 or 1

3. Find a so that the quadratic equation $(a-1)x^2 + 3x - a + 2 = 0$ has a solution $x = -1$.

   (a) No such a **(b)** $a = 0$ **(c)** $a = 1$ **(d)** $a = 2$

   (e) None of the above

4. A group of 50 high school math concentration students are comparing how many of their three different math exams (Algebra, Geometry and Calculus) they passed at the first attempt. You are given the following information:

   1. 56% of students passed Geometry at the first attempt.

   2. 58% of students passed Calculus at the first attempt.

   3. 19 students passed both Geometry and Calculus at the first attempt.

   4. 2 students did not pass any exam at the first attempt.

   Calculate how many students passed only the Algebra exam at the first attempt.

   (a) 3 **(b)** 5 **(c)** 10

   (d) 24 **(e)** Not enough information

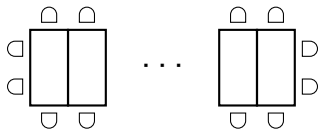
5. The Bathula family has 6 sons. Each son has 3 sisters. How many children are there?

   (a) 37 **(b)** 9 **(c)** 36 **(d)** 18 **(e)** 24

6. A snail at the bottom of a well goes up 10 feet each day and slides back 5 feet at night. How many days does it take the snail reach the top of the well if it is 40 feet deep?

   (a) 4 **(b)** 5 **(c)** 3 **(d)** 8 **(e)** 7

7. I am inviting some people to a party, and I want to seat them all at one long table. I want to put together a series of rectangular tables, and form one long table, as shown. What is a formula that relates the number of chairs (C) to the number of tables (T)?



   (a) $C = 2T + 4$ **(b)** $C = 2T$ **(c)** $C = 4T + 2$ **(d)** $C = T + 4$ **(e)** $C = T$

8. The set A has c members, and the set B has f members, where $c \geq f$. What is the largest number of members $A \cup B$ i.e., the set of elements that are in A or B, could have?

   (a) $c - f$ **(b)** $c \cdot f$ **(c)** $c + f$ **(d)** f **(e)** c

9. Find all possible ways to fill in the missing digits so that the number 546,5__ is divisible by 2 and 5 but not by 3.

   (a) 10, 20, 30, 40, 50, 60, 70, 80, 90 **(b)** 00, 20, 30, 50, 60, 80

   (c) 00, 10, 30, 50, 70, 90 **(d)** 00, 20, 30, 50, 60, 80, 90

   (e) 20, 30, 50, 60, 80, 90

10. What base-eight numeral follows 37_{eight}?

    (a) 37_{eight} **(b)** 47_{eight} **(c)** 38_{eight} **(d)** 100_{eight} **(e)** 40_{eight}

11. Convert base-ten numeral 15 to base eight.

    (a) 16_{eight} **(b)** 10_{eight} **(c)** 12_{eight} **(d)** 17_{eight} **(e)** 13_{eight}

12. Two buses leave the terminal at 8 A.M. Bus number 36 takes 85 minutes to complete its route; bus number 86 takes 102 minutes. When is the next time the two buses will arrive together at the terminal (assuming they are on time and they spend no time waiting between routes)?

    (a) in 510 min **(b)** in 102 min **(c)** in 289 min **(d)** in 408 min

    (e) in 8,670 min

13. Let $a = 2^5 \cdot 5^3 \cdot 11^2$. Let b be such that the greatest common factor of a and b is $2 \cdot 5^3 \cdot 11$ and the least common multiple of a and b is $2^5 \cdot 3^3 \cdot 5^3 \cdot 11^2$. Find b.

    (a) $b = 2 \cdot 3^3 \cdot 5^2 \cdot 11^2$ **(b)** $b = 2^5 \cdot 3^3 \cdot 5^3 \cdot 11^2$ **(c)** $b = 2^5 \cdot 3^5 \cdot 5^6 \cdot 11$

    (d) $b = 2 \cdot 3^3 \cdot 5^3 \cdot 11$ **(e)** $b = 2 \cdot 3 \cdot 5 \cdot 11$

14. If a is 50% more than b, then b is how many percent of a? Please round the answer to the nearest tenth.

    (a) 71.7 **(b)** 56.7 **(c)** 66.7 **(d)** 67.7 **(e)** 64.7

15. If the repeating decimal $0.84\overline{51}$ is represented by the fraction $\frac{a}{b}$, where a and b are positive integers with no common factors greater than 1, find $a + b$.

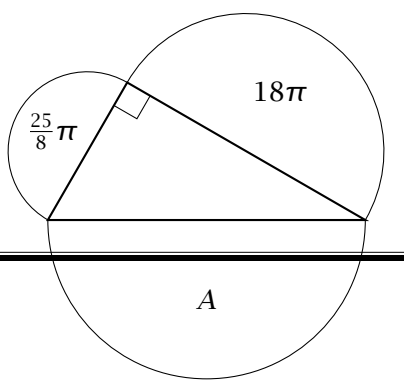
    (a) 303 **(b)** 4617 **(c)** 5211 **(d)** 6089 **(e)** 8451

16. The area of a circle is 36π m^2. What is the exact circumference?

    (a) 12π m **(b)** 12 m **(c)** 6π m **(d)** 6 m **(e)** 36π m

17. The areas of the two smaller semicircles are $\frac{25}{8}\pi$ and 18π. Find the area of the largest semicircle.

    (a) $\frac{169}{8}\pi$ **(b)** 25π **(c)** $\frac{25}{8}\pi$ **(d)** 50π **(e)** $\frac{289}{8}\pi$



18. Joe has three drinking cups: a cylinder-shaped, a cone-shaped and a semi-sphere-shaped. All of them have the same radius and height. The cylinder-shaped cup is full of water. Joe want to pour the water from the cylinder-shaped cup to the other two cups. Which of the following is going to happen?

    (a) There will not be enough water to fill the two cups.

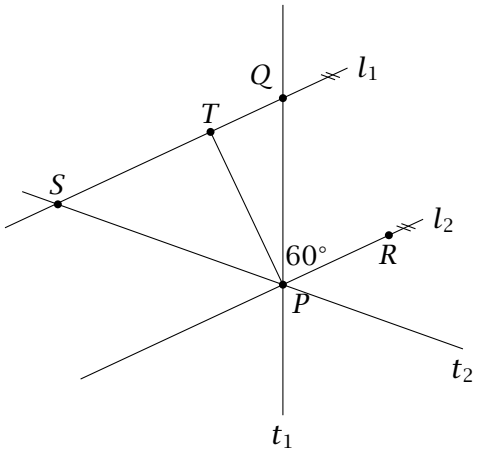
    (b) The water from the cylinder-shaped cup will exactly fill the two other cups.

    (c) There will be too much water to fit into the other two cups.

    (d) It depends on what the radius and height are.

    (e) It depends on what the radius is.

19. Given that l_1 and l_2 are parallel and t_1 and t_2 and are transversals that intersect l_2 at P. Additionally, $\overline{PT}$ is perpendicular to l_1, $m\angle QPR = 60°$, $PT = TS$ and $PQ = 5$. What is the length of $\overline{PS}$?



    (a) $\frac{5\sqrt{3}}{2}$ **(b)** $\frac{\sqrt{6}}{2}$

    (c) $\frac{25\sqrt{3}}{2}$ **(d)** $\frac{5\sqrt{6}}{2}$ **(e)** None of the above

20. David, Bill and George are three thieves. One of them committed a robbery. During the interrogation they made the following statements:

    – David: Bill is not the robber. George is the robber.

    – Bill: David is innocent. George is the robber.

    – George: I am innocent. David is the robber.

    It was determined that one of them lied twice, one of them told the truth twice, and one lied once and told the truth once. Who is the robber?

    (a) David **(b)** Bill **(c)** George

    (d) None of them **(e)** Impossible to determine

21. Suppose $f(x) = ax + b$. If $f(f(f(x))) = 64x + 63$, what is $a + b$?

(a) 3 **(b)** 7 **(c)** 8 **(d)** 12 **(e)** None of the above

22. For which values of k does the system

$$x^2 - y^2 = 0$$
$$(x - k)^2 + y^2 = 2$$

have exactly two solutions of the form (x, y) where x and y are real numbers?

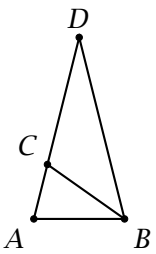
(a) $k = \pm 1$ **(b)** $k = \pm 2$ **(c)** $k = 1$ and $k = -2$

(d) $k = -1$ and $k = 2$ **(e)** None of the above

23. A library is open every day except Sunday. Max, Gus and Zyk visit the library together for the first time. After the first visit, Max always visits the library two days after her previous visit, except when the library is closed, in which case she goes 3 days after her previous visit. Gus always visits the library three days after his previous visit, except when the library is closed, in which case he goes 4 days after his previous visit. Zyk always makes his next visit 4 days after the previous visit, unless the library is closed, in which case he goes 5 days after his previous visit. If the second time they all visit the library together falls on Friday, what day of the week was their first visit?

(a) Monday **(b)** Tuesday **(c)** Wednesday **(d)** Thursday **(e)** Friday

24. In the triangle shown on the right, $AB = BC = CD$ and $AD = BD$. Find the measure of angle D.



(a) 28° **(b)** 30° **(c)** 36° **(d)** 72° **(e)** None of the above

25. Two particles move clockwise around a circle with circumference 300 feet. The faster particle moves at a constant speed of R feet per second, and the slower particle moves at a constant speed of r feet per second. If the particles meet every 50 seconds, then what is the value of $R - r$ in feet per second?

(a) 6 **(b)** 8 **(c)** 10 **(d)** 12 **(e)** 14