

Saginaw Valley State University
2009 Math Olympics – Level I

1. A man and his wife take a trip that usually takes three hours if they drive at an average speed of 60 mi/h. After an hour and a half of driving at a steady rate of 60 mi/h they stop for a 45 minute lunch break. How fast do they need to drive on the rest of the trip to arrive three hours after they started?

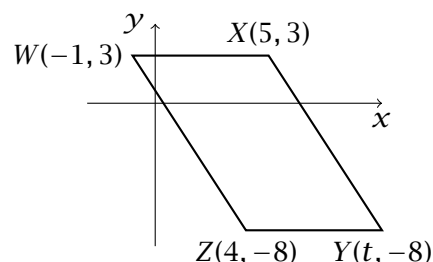
(a) 90 mi/h (b) 120 mi/h (c) 80 mi/h (d) 105 mi/h (e) None of the above

2. Ali and Fred are driving to Bing’s house. Ali confuses left with right 50% of the time (This means half the time, when he means to say “left” he says “right” instead, and half the time when he means to say “right” he really says “left”). Fred gets left and right confused 25% of the time. (This means when someone tells him to turn left, 75% of the time he will turn left, and 25% he will turn right instead, and when someone tells him to turn right, 75% of the time he will turn right, and 25% he will turn left instead.) If Fred is driving and Ali is giving directions, and there are two left turns on the way to Bing’s house (and no other turns) what is the probability they will get there?

(a) $\frac{1}{4}$ (b) $\frac{9}{64}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) None of the above

3. If $WXYZ$ is a parallelogram, the t equals:

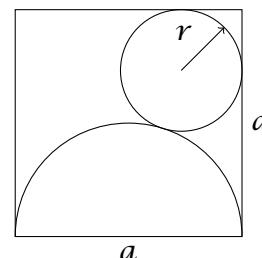
(a) 8 (b) 9 (c) 10 (d) 11 (e) 12



4. The symbol R_k stands for a positive integer whose base-ten representation is a sequence of k ones, that is $R_1 = 1$, $R_2 = 11$, $R_3 = 111$, etc. The quotient $\frac{R_{24}}{R_4}$ is an integer whose base-ten representation contains only digits 0 and 1. The number of digits 0 in this representation is:

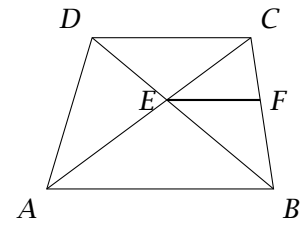
(a) 6 (b) 13 (c) 15 (d) 16 (e) 20

5. A square with side a has a semicircle constructed inside it such that the diameter of the semicircle is one of the sides of the square. A circle with maximal radius is then constructed inside the square, but outside of the semicircle (see illustration). Express the radius r of the circle in terms of a .



(a) $r = a(2 - \sqrt{3})$ (b) $r = a/4$
(c) $r = a(3 - \sqrt{2})$ (d) $r = \sqrt{a}/2$ (e) None of the above

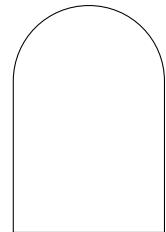
6. In the trapezoid $ABCD$ (the picture is not drawn to scale), the sides AB and CD are parallel, and $|AB| = 4$ and $|CD| = 3$. The point E is the intersection point of the two diagonals, and the segment EF is parallel to AB and CD . The length of EF is



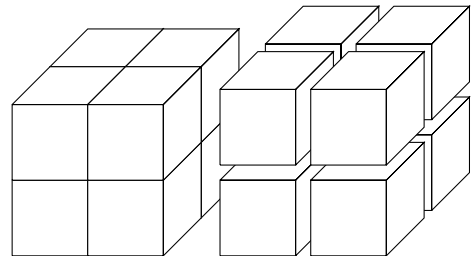
- (a) 3
- (b) 6
- (c) $12/7$
- (d) $3/2$
- (e) Not enough information given
7. A company offers three types of benefits, A, B, and C. As part of this plan, the individual employees may choose either exactly two benefit types, or no benefits at all. The proportion of the company's employees that choose benefit types A, B and C are $1/4$, $1/3$ and $5/12$, respectively. Determine the probability that a randomly chosen employee will choose no benefits at all.

- (a) 0 (b) $47/144$ (c) $1/2$ (d) $97/144$ (e) $7/9$

8. The (possibly out-of-scale) picture shows a window whose shape consists of a semicircle on top of a square. If the perimeter of the window is 20 ft, what is the exact length of the bottom side of the window?



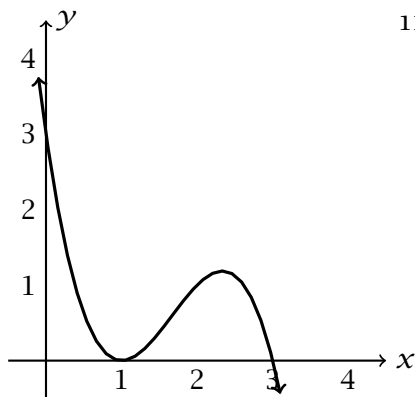
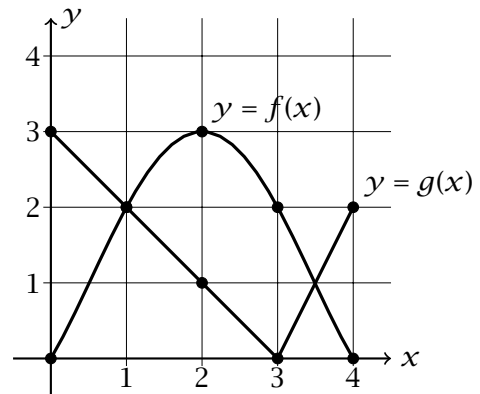
- (a) $\frac{40}{\pi+3}$ ft
- (b) $\frac{40}{\pi+6}$ ft (c) $\frac{20}{2\pi+3}$ ft (d) $\frac{20}{\pi+3}$ ft (e) None of the above
9. A rectangular parallelepiped is cut into 8 smaller rectangular parallelepipeds by three cuts parallel to the sides. The 8 smaller solids are then separated, as shown in the picture. What is the percent increase in the total surface area?



- (a) 25% (b) 33% (c) 50% (d) 75%
- (e) 100%

10. Find $(g \circ f)(1)$.

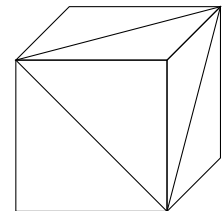
- (a) 1 (b) 2
 (c) 3 (d) 4 (e) Not enough information given

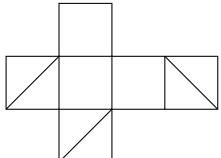
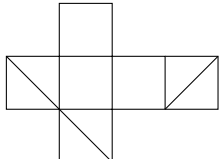
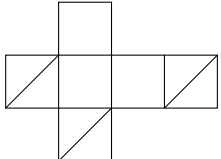
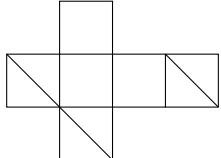


11. The picture shows the graph of a third degree polynomial $f(x)$. Which of the following gives $f(x)$?

- (a) $f(x) = (x - 1)(x - 3)^2$
 (b) $f(x) = (x - 1)^2(x - 3)$ (c) $f(x) = (1 - x)^2(3 - x)$
 (d) $f(x) = (1 - x)(3 - x)^2$ (e) $f(x) = (x^2 + 1)(x - 3)$

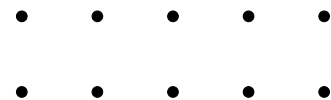
12. On three sides of a cube, diagonals are drawn as shown in the picture. If we unfold the cube, which of the following could we obtain?



- (a)  (b) 
 (c)  (d) 

(e) None of the above

13. The ten points on the right form a square grid. How many isosceles triangles can be drawn with vertices at the points?



- (a) 8 (b) 16 (c) 24 (d) 32 (e) 40

14. The numbers from 1 to 100 are written onto slips of paper and placed into a hat. How many numbers do we have to pull out of the hat to make sure that their product is divisible by 10?

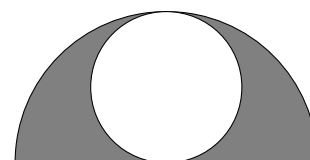
(a) 52 (b) 80 (c) 81 (d) 82 (e) 91

15. Simplify

$$\sqrt{\frac{4 + \sqrt{15}}{4 - \sqrt{15}}} + \sqrt{\frac{4 - \sqrt{15}}{4 + \sqrt{15}}}$$

(a) 0 (b) 2 (c) 8 (d) 19 (e) None of the above

16. In the drawing on the right, the small white circle is the largest circle that fits inside the semicircle. What is the ratio of the area of the small circle to the shaded area?



(a) 2 : 3 (b) 1 : 1 (c) 1 : 2 (d) 3 : 4 (e) 2 : π

17. What is 5% of 25% of 4000?

(a) 32 (b) 50 (c) 125 (d) 200 (e) 500

18. If $a * b$ is the greatest common factor of two positive integers a and b , how many of the following equations are *not* always true?

$$a * 1 = a$$

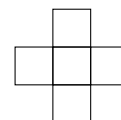
$$a * b = b * a$$

$$a * (b * c) = (a * b) * c$$

$$a * (b + c) = a * b + a * c$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

19. Suppose we arrange the numbers 1, 2, 3, 4 and 5 in the five squares so that the horizontal and the vertical line both add up to 8. Which number has to go in the middle square?



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

20. For integers a , b and c , define $(a \uparrow b \downarrow c)$ to mean $a^b - b^c + c^a$. Then $(1 \uparrow -1 \downarrow 2)$ equals

(a) -4 (b) -2 (c) 0 (d) 2 (e) 4

21. A frog starts out by hopping one inch. Each successive hop is 2 inches longer than the previous hop. How far has he gone all together at the end of his 13th hop?

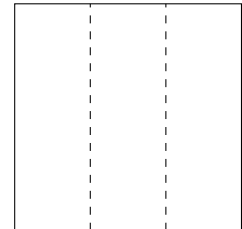
- (a) 25 in (b) 169 in (c) 91 in (d) 157 in (e) None of the above

22. Four points are on a line segment, as shown. If $|AB| : |BC| = 1 : 2$ and $|BC| : |CD| = 8 : 5$, then $|AB| : |BD|$ equals



- (a) 4 : 13 (b) 1 : 13 (c) 1 : 7 (d) 3 : 13 (e) 4 : 17

23. A square is cut into three congruent rectangles along two lines parallel to a side, as shown. If the perimeter of each of the three rectangles is 24, then the area of the original square is



- (a) 24 (b) 36 (c) 64 (d) 81 (e) 96

24. Which of the following triangles cannot exist?

- (a) An acute isosceles triangle (b) An isosceles right triangle
(c) An obtuse right triangle (d) A scalene right triangle
(e) A scalene obtuse triangle

25. Friday the 13th occurred two months in a row this year. What is the next year that this will happen?

- (a) 2021 (b) 2016 (c) 2015 (d) 2014 (e) None of the above
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