

Saginaw Valley State University  
2005 Math Olympics - Level I

1) Which of the following numbers is a prime?

- a) 2783      b) 637      c) 247      d) 359      e) none of the above

2) Simplify the expression.  $\frac{x^2 - y^2}{x^{-1} - y^{-1}}$

- a)  $(x - y)^2(x + y)$     b)  $-x^2y - xy^2$     c) 247    d)  $x + y$     e) none of the above

3) When going outside on a cold winter day, Jill can choose from three winter coats, five wool scarves, two pairs of boots and four hats. The coats are of distinct colors, and so are the scarves. Jill will never wear her green scarf unless she wears the blue coat. If Jill wears a coat, a scarf, boots and a hat, how many different outfits might her friends see her in?

- a) 120      b) 104      c) 97      d) 119      e) none of the above

4) Coffee costs \$6 per pound in the United States. If a Canadian dollar exchanges for 75 U.S. cents, and a kilogram is 2.2 pounds, what is the equivalent cost in Canadian dollars per kilogram?

- a) \$17.60      b) \$9.90      c) \$3.64      d) \$2.05      e) none of the above

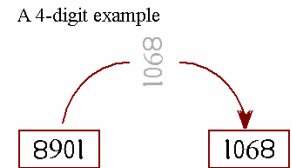
5) A table in a doll house is a scale model of a full sized table. The surface area of the top of the full sized table is 800 square inches. The surface area of the top of the doll house table is 2 square inches. If the doll house table is 1.5 inches high, how high is the full sized table?

- a) 30 in      b) 45 in      c) 40 in      d) 28 in      e) none of the above

6) Solve the equation.  $\left(\frac{1}{3}\right)^{5-x} = 9 \cdot 9^x$

- a)  $x = \frac{5}{3}$     b)  $x = -7$     c) no solution    d)  $x = -5$     e) none of the above

- 7) Assume that the digits 1, 0, and 8 are written so that they look the same when they are upside down as they do when they're right-side up. Assume that the digits 6 and 9 are written so that by turning (rotating) a 6 upside down, we get a 9. How many five-digit numbers, not including those which start with zero, look the same when rotated upside down as they do right-side up?



- a) 75    b) 868    c) 152    d) 48    e) none of the above

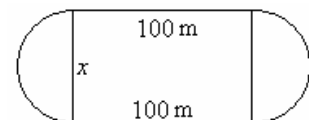
- 8) A class has 20 students. If every student shakes hands with every other student, how many handshakes will there be?

- a) 190    b) 400    c) 380    d)  $20^{19}$     e) none of the above

- 9) A school has three clubs: Cheerleaders, Pep Club and Student Council. Half of the student council members and all of the cheerleaders belong to the pep club. Exactly two students are members of all three clubs. If there are 24 student council members, 6 cheerleaders, and 40 pep club members, how many students belong only to the pep club?

- a) 10    b) 24    c) 20    d) 22    e) none of the above

- 10) A 400 meter track is to be built with 100-meter straight-aways on both sides and semicircular ends. (See the diagram.) How many meters wide,  $x$ , should the track be between the straight-aways so that the total perimeter is 400 meters?



- a)  $100/\pi$     b) 100    c)  $200/\pi$     d)  $50/\pi$     e) none of the above

11) Find the perimeter of a rectangle whose area is 22 square meters and whose diagonal is 10 meters.

- a) 24      b) 12      c) 44      d) 33      e) none of the above

12) Donald Duck can eat 2 pizzas in 3 minutes, while Goofy can eat 3 pizzas in 2 minutes. At these rates, how many pizzas can they eat together in one hour?

- a) 54      b) 96      c) 130      d) 216      e) 250

13) If  $m > 0$  and the points  $(m, 3)$  and  $(1, m)$  lie on a line with slope  $m$ , then  $m =$

- a) 1      b)  $\sqrt{2}$       c)  $\sqrt{3}$       d)  $\sqrt{5}$       e) none of the above

14) An urn is filled with coins and beads, all of which are either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percent of the objects in the urn are gold coins?

- a) 40%      b) 48%      c) 52%      d) 60%      e) none of the above

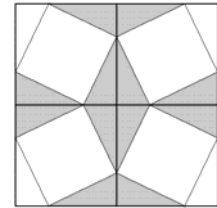
15)  $\sqrt{\frac{8^{10} + 4^{10}}{8^4 + 4^{11}}} =$

- a)  $\sqrt{2}$       b) 16      c) 32      d)  $12^{\frac{2}{3}}$       e) none of the above

- 16) The ratio of the radii of two concentric circles (circles having the same center) is 1:3. If  $\overline{AC}$  is a diameter of the larger circle,  $\overline{BC}$  is a chord of the larger circle that is tangent to the smaller circle, and  $AB = 12$ , then the radius of the larger circle is

a) 13            b) 18            c) 21            d) 24            e) none of the above

- 17) The large  $12\text{in} \times 12\text{in}$  square shown to the right is made up of four square tiles each having a small square inscribed in it. The four tiles are congruent, and some are mirror images of the others. The shaded region has area  $24\text{ in}^2$ . The side length of each of the small inscribed squares is



a)  $2\sqrt{5}$             b)  $2\sqrt{3}$             c) 3            d)  $\sqrt{28}$             e) none of the above

- 18) At a high school each freshman is taking exactly two of the following three courses: English, Math, Social Studies. The freshmen enrollments are: 20 in English, 17 in Math, 11 in Social Studies. How many freshmen are there?

a) 24            b) 25            c) 26            d) 27            e) none of the above

- 19) Let a polynomial  $P$  be a degree four polynomial such that  $0 = P(0) = P(1) = P(2) = P(-1)$  and  $P(-2) = 12$ . Then  $P(3)$  equals

a)  $-12$             b)  $-1/2$             c) 4            d) 6            e) none of the above

- 20) Jody read a book in 3 days. During the first day she read  $1/5$  of the book, plus 16 pages. During the second day she read  $3/10$  of what remained, plus 20 pages. During the third day she read  $3/4$  of what remained, plus 30 pages. How many pages were there in the book?

a) 225            b) 240            c) 265            d) 270            e) none of the above

21) Determine a, b for which  $(-4, -3)$  is a solution of the system

$$ax + by = -26$$

$$bx - ay = 7$$

a)  $a = -2, b = 11$

b)  $a = 2, b = -6$

c)  $a = 9, b = 5$

d)  $a = 5, b = 2$

e) none of the above

22) The Jones family is remodeling their house. The width of a rectangular room is increased by 30% and the length of that room is increased by 25%. By what percentage is the area of the room increased?

a) 55%

b) 62.5%

c) 7.5%

d) 75%

e) none of the above

23) Which of these numbers is the smallest?

a)  $\frac{\sqrt{9^3}}{\sqrt[3]{27^{15}}}$

b)  $\sqrt{\sqrt[3]{3^{-60}}}$

c)  $1/12$

d)  $\frac{4^2 2^{-4}}{3^{11}}$

e)  $\frac{9^{-1}}{27 \cdot 3^5}$

24) The three circles have radii 1in, 2in and 3in. What fraction of the area of the outer square is shaded?

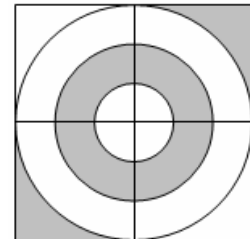
a)  $18 - \frac{3\pi}{2}$

b)  $\frac{3}{8}$

c)  $\frac{1}{2} - \frac{\pi}{24}$

d)  $\frac{5\pi}{6}$

e) none of the above



25) A parallelogram ABCD is drawn so that the triangle ABC is an isosceles triangle inscribed in a circle of radius 12, so that the side AB passes through the center. What is the area of the parallelogram?

a) 288

b) 36

c) 72

d) 144

e) None of the above

