

Saginaw Valley State University  
2005 Math Olympics - Level II

1) If the point  $(3, 5)$  is on the graph of  $y = -3f(2x + 1) + 2$ , which of the following must be true?

- a)  $f(7) = 13$       b)  $f(2) = 13$       c)  $f(2) = -1$       d)  $f(7) = -1$   
e) None of the preceding

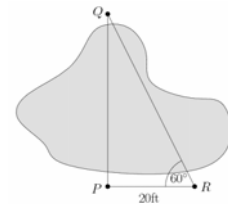
2) Let  $p$  be a prime number and  $k$  an integer such that  $x^2 + kx + p = 0$  has two distinct positive integer solutions. The value of  $k + p$  is

- a) 1      b) -1      c) 0      d) 2      e) -2

3) Which of the following are equivalent to  $\log(\csc^2 x)$ ?

- a)  $\frac{1}{\log^2(\sin x)}$       b)  $\frac{1}{\log^2(\cos x)}$       c)  $-2\log(\sin x)$   
d)  $\log(2 \csc x)$       e) None of the preceding

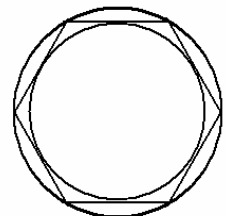
4) To find the distance between two points  $P$  and  $Q$  on opposite sides of a lake, a surveyor locates a point  $R$ , 20 feet from  $P$ , such that the line  $PR$  is perpendicular to the line  $PQ$ . The angle  $PRQ$  is  $60^\circ$ . Find the distance between  $P$  and  $Q$ .



- a)  $20\sqrt{3}$  ft      b) 40ft      c)  $\frac{20\sqrt{30}}{3}$  ft  
d) 30 ft      e) None of the preceding

5) The difference between the area of the circle inscribed (the inner circle) in a regular hexagon of side length  $\sqrt{3}$  and the area of the circle in which the hexagon is inscribed (the outer circle) is

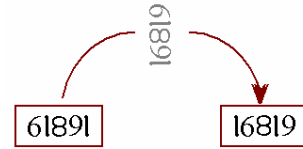
- a)  $\pi/2$       b)  $3\pi/2$       c)  $3\pi/4$       d)  $2\pi/3$       e) None of the preceding



- 6) The product  $N$  of three positive integers is six times their sum, and one of the integers is the sum of the other two. Find the sum of all possible values of  $N$

a) 300    b) 350    c) 336    d) 318    e) None of the preceding

- 7) Suppose that the digits 1, 0, and 8 are written so that they look the same when they are upside down as they do when they're right-side up. Also, the digits 6 and 9 are written so that by turning (rotating) a 6 upside down, we get a 9. Two 5-digit numbers are "flips" of each other if one is obtained by rotating the other upside down. For example, 61891 and 16819 are "flips" of each other. If all five-digit numbers (including those starting with 0 like 00027) are being printed on slips of paper such that two numbers are printed on the same slip if and only if they are flips of one another, how many slips of paper are needed?



a) 100,000                      b) 98475                      c) 96875  
 d)  $10^5 - \frac{5^5}{2}$                       e) None of the preceding

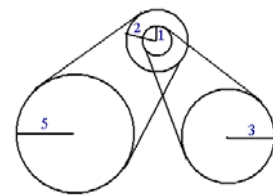
- 8) Which of the following is **not** equivalent to  $\sin(15^\circ)$ ?

a)  $\frac{\sqrt{6} - \sqrt{2}}{4}$     b)  $\frac{\sqrt{2} - \sqrt{3}}{2}$     c)  $\frac{1}{\sqrt{6} + \sqrt{2}}$     d)  $\frac{2 - \sqrt{3}}{4}$     e)  $\frac{\sin 75^\circ - \sqrt{3} \cos 75^\circ}{2}$

- 9) Let  $x \odot y = \frac{x}{y} + \frac{y}{x}$ . If  $a$  is a positive number such that  $a \odot (a+1) = \frac{5}{2}$ , which of the below gives the value of  $a$ ?

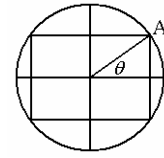
a) 1                      b) 2                      c)  $\sqrt{5}$                       d) there is no such number  
 e) None of the preceding

- 10) A belt driven wheel system shown in the figure consists of two outside simple wheels of radii 5in and 3in, respectively, and a middle compound wheel made by gluing concentric 2in and 1in wheels, so that the two wheels will rotate as one unit. Two belts are attached tightly between the wheels as shown. If the 5in wheel is moving at 4 rotations per minute (rpm), how fast is the 3in wheel rotating?



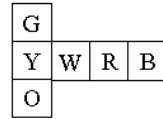
a) 20/3 rpm    b) 20 rpm    c) 24 rpm    d) 30 rpm    e) None of the preceding

11) A rectangle is inscribed in a circle of radius 1. The angle from the center to the corner A is  $\theta$ . The area of the rectangle equals



- a)  $\cos^2 \theta - \sin^2 \theta$       b)  $\sin 2\theta$       c)  $2 \sin 2\theta$   
 d)  $2 \sin \theta \cos \theta$       e) None of the preceding

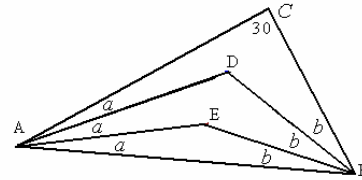
12) A butterfly lands on one of the six squares of the t-shaped figure shown and then randomly moves to an adjacent square. What is the probability that the butterfly ends up on the R square?



(Here, two squares are adjacent if and only if they share a side)

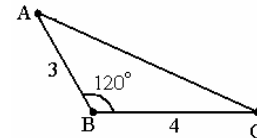
- a) 1/2      b) 1/3      c) 1/4      d) 1/6      e) None of the preceding

13) The corresponding tri-sector of two angles (A and B) of a scalene triangle ABC meet at points D and E. The third angle of the triangle (angle C) is 30 degrees. Find the measure of angle ADB.



- a)  $130^\circ$       b)  $80^\circ$       c)  $120^\circ$       d)  $90^\circ$       e) None of the preceding

14) In  $\triangle ABC$ ,  $m\angle ABC = 120^\circ$ ,  $AB = 3$  and  $BC = 4$ . If perpendiculars constructed to  $\overline{AB}$  at A and to  $\overline{BC}$  at C meet at D, then  $CD =$



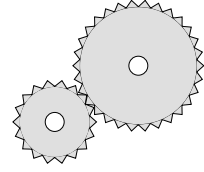
- a) 5      b)  $\frac{8}{\sqrt{3}}$       c)  $\frac{11}{2}$       d)  $\frac{10}{\sqrt{3}}$       e) None of the preceding

15) A square of perimeter 20 is inscribed in a square of perimeter 28 (inscribed means the vertices of the smaller square are on the sides of the larger square). What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

- a)  $5\sqrt{3}$       b)  $\sqrt{58}$       c)  $\frac{7\sqrt{5}}{2}$       d)  $\sqrt{65}$       e) None of the preceding

- 16) The larger gear has 25 teeth, and the smaller has 15 teeth. If the larger gear rotates through an angle of  $90^\circ$ , through what angle measure does the small gear rotate?

(The picture illustrates how the wheels move together but it's not accurate in terms of the number of teeth!)



- a)  $54^\circ$                       b)  $150^\circ$                       c)  $90^\circ$   
 d)  $180^\circ$                       e) None of the preceding
- 17) The Beatles (Paul, John, George and Ringo) took a math exam. Paul got correct half of the questions plus 7 questions, John got correct one third of the questions plus 17 questions, George got correct one fourth of the questions plus 22 questions, and Ringo got correct one fifth of the questions plus 25 questions. There were between one and 100 questions on the exam and each Beatle got an integer number of questions correct. Which Beatle got the most questions correct?
- a) Only Paul                      b) Only John                      c) Only George                      d) Only Ringo  
 e) At least two Beatles got the most questions correct

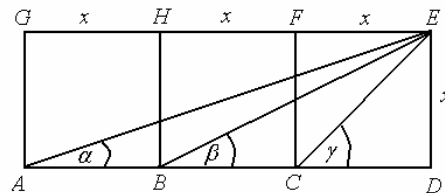
- 18) If  $L$  is the line whose equation is  $ax + by = c$ . Let  $M$  be the reflection of  $L$  through the  $y$ -axis, and let  $N$  be the reflection of  $L$  through the  $x$ -axis. Which of the following must be true about  $M$  and  $N$  for all choices of  $a$ ,  $b$ , and  $c$ ?

- a) The  $x$ -intercepts of  $M$  and  $N$  are equal  
 b) The  $y$ -intercepts of  $M$  and  $N$  are equal  
 c) The slopes of  $M$  and  $N$  are equal  
 d) The slopes of  $M$  and  $N$  are negative reciprocals of each another  
 e) None of the preceding

- 19) Let  $f(x)$  be a cubic polynomial in one variable  $x$  such that  $f(0) \neq 0$  and with no repeated roots. Suppose that  $f(x)$  and  $f(-x)$  share a common root. Which of the following polynomials could be the least common multiple (LCM) of  $f(x)$  and  $f(-x)$ ? (The numbers  $\pm a$ ,  $\pm b$  and  $\pm c$  below are all distinct)

- a)  $(x^2 - a^2)^2(x - b)(x - c)$                       b)  $(x^2 - a)^2(x^2 - b)$                       c)  $(x^2 - a^2)(x - b)(x - c)$   
 d)  $(x^2 - a^2)(x^2 - b^2)$                       e) None of the preceding could be the LCM

- 20) In the figure to the right, ABHG, BCFH and CDEF are congruent squares with side length  $x$ . The sum of the angles  $\alpha + \beta$  equals



- a)  $\pi/3$                       b)  $3\alpha$                       c)  $\gamma$                       d)  $\pi/2$                       e) None of the preceding

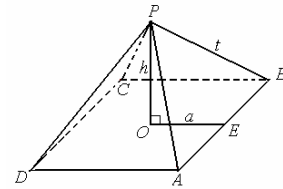
21) The three little pigs are digging a moat to keep nasty wolves away. The first two pigs, working together, could dig the moat in two hours. The first and third pigs, working together, could dig the moat in one hour and twelve minutes. The second and third pigs, working together, could complete the job in an hour and a half. How long will it take all three pigs working together to dig the moat?

- a) 36 min      b) 45 min.      c) 50 min.      d) 54 min.      e) One hour.

22) Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 ft and rotates at a constant rate of one revolution per minute. How many seconds does it take a rider travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

- a) 5      b) 6      c) 7.5      d) 10      e) None of the preceding

23) For the right regular pyramid with a square base shown in the figure to the right, the ratio  $h : a$  is  $\sqrt{3} : 1$ , find the ratio  $t : a$



- a) 2 : 1      b)  $2\sqrt{2} : 1$       c) 4:1  
d)  $\sqrt{5} : 1$       e) None of the preceding

24) Roadrunner and Coyote are running on a highway simultaneously starting at the same point. Each maintains a constant speed within each 4-mile interval of the road. The speed of Coyote doubles from one interval to the next, while the speed of Roadrunner is reduced by half after each interval. Roadrunner starts at the speed of 32 mph and Coyote at 1 mph. By the time Coyote catches up with Roadrunner, how many (complete) miles would each have run?

- a) 20 mi      b) 22 mi      c) 24 mi      d) 26 mi      e) None of the preceding

25) The vertex of the parabola that passes through the points  $(-1,0)$ ,  $(0,1)$  and  $(1,3)$  is

- a)  $(-\frac{3}{2}, -\frac{1}{8})$       b)  $(\frac{3}{2}, 8)$       c)  $(-2, -1)$       d)  $(\frac{1}{2}, \frac{3}{2})$   
e) None of the preceding