Naturally, there are alternative ways to do each of the problems. We are providing one.

1) If A is 15% of C and B is 25% of C, then what percent of B is A?

a) 75 b) 120 c) 60
d) 175 e) None of the above

Solution: c) 60
B = 0.25C implies C = (1/0.25) B. Now A = 0.15C = 0.15(B/0.25) =0.60B. So, A = 60% of B

2) It took Tim 30 minutes to walk from his home to his friend's house to pick up his bicycle. On the bike Tim is 5 times faster than on foot. How long will it take Tim to ride back home if he chooses a route back that is twice as long as the one he took walking from home?

a) 18 min b) 12 min c) 6 min
d) 15 min e) None of the above

Solution: b) 12 min
Let t be the time of the return trip. In the table, v denotes Tim's speed on foot. Equating the distance of the return trip to twice the distance of the trip out we get $5vt = 2\times(30v)$. So $t = 12$ min.

Trip	Time (min)	Speed	Distance = time× speed
Out	30	v	$30v$
Return	t	$5v$	$5vt$

3) The value of $\frac{99\times 101}{0.10}$ is closest to

a) 100 b) 1,000 c) 10,000 d) 100,000 e) 1,000,000

Solution: d) 100,000

$$\frac{99\times 101}{0.10}\approx\frac{100\times 100}{0.10}=100{,}000$$

4) Which of the following is largest

a) 4^{40} b) 8^{26} c) 2^{76}

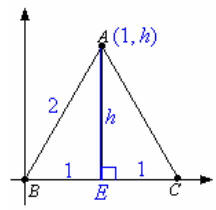
d) $16^{18}\times 2^{4}$ e) $\frac{4^{42}+16(32)^{16}}{64}$

Solution: a) 4^{40}
Writing all numbers as powers of 2 we have: a) $4^{40}=2^{80}$, b) $8^{26}=(2^3)^{26}=2^{78}$, c) 2^{76}, d) $(16)^{18}\text{ x }2^4=(2^4)^{18}\text{ x }2^4=2^{76}$, and

e) $$\frac{4^{42}+16(32)^{16}}{64}=\frac{2^{84}+2^4(2^5)^{16}}{2^6}=\frac{2^{84}+2^{84}}{2^6}=\frac{2(2^{84})}{2^6}=\frac{2^{85}}{2^6}=2^{79}$$

Therefore the largest is 4^{40}

5) In the picture, the triangle $\triangle ABC$ is equilateral with side length 2 and the vertex B at the origin of the Cartesian plane. The coordinates of the vertex A are



a) $(2, \sqrt{3})$ b) $(1, \sqrt{3})$ c) $(2, \sqrt{3}/2)$
d) (1,2) e) None of the above

Solution: b) $(1, \sqrt{3})$
Since the triangle ABC is equilateral, the segment from A to the midpoint E of the segment BC is perpendicular to the side BC. So the coordinates of A are (1, h), where h is the length of height AE. In the right triangle AEB, $h = \sqrt{2^2 - 1^2} = \sqrt{3}$. Therefore A = $(1, \sqrt{3})$

6) Let A and B be two circles such that the circumference of A is 10% of that of B. If the area of A is 5, what is the area of B?

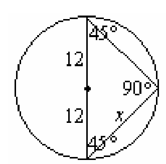
a) 50 b) 50π c) $50\pi^2$ d) $500\pi^2$ e) None of the above

Solution: e) None of the above
If r is the radius of the circle B, then the circumference of B is $2\pi r$. So, the circumference of the circle A is $0.1(2\pi r) = 2\pi(0.1r)$. In particular, A has radius $0.1r$. Now 5 = Area of A = $\pi(0.1r)^2$ implies $r^2 = 5/0.01\,\pi$. Therefore the area of the circle B = $\pi r^2 = \pi(5/(0.01\pi)) = 500$ units. [*Alternative solution:* The circles are (of course) geometrically similar. The linear scaling factor from A to B is 10. The areas are scaled by the square of the linear scaling factor; hence the area is scaled by factor 100. Since the area of A is 5, it follows that the area of B should be 500.]

7) An isosceles triangle is inscribed inside a circle of radius 12 so that one of its sides passes through the center. What is the area of the triangle?

a) 18 b) 36 c) 72 d) 144 e) 288

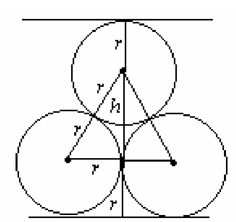
Solution: d) 144
The triangle is a right triangle. The side through the center is the hypotenuse and is a diameter of the circle, hence has length 24. Let x be the length of the (equal) legs of the triangle. Area = $\frac{1}{2}x^2$. Using Pythagorean Theorem: $x^2 + x^2 = 24^2$. Solving for x^2 gives $x^2 = 288$. So the area is 144.



8) Three circles of the same radius r are stacked as in the figure. The height of the stack in terms of the radius r is

a) $2r$ b) $3r$ c) $\sqrt{3}r$
d) $(2+\sqrt{3})r$ e) $2\sqrt{3}r$

Solution: d) $(2+\sqrt{3})r$
The height of the stack is $2r + h$ where h is the height of the equilateral triangle of side length $2r$ created by connecting the centers of the three circles (as shown). h is found using the Pythagorean Theorem: $r^2 + h^2 = (2r)^2$; giving $h = r\sqrt{3}$.



9) Let the operation $*$ be defined as $a * b = b + \frac{1}{a}$. The value of $(1 * 2) * 4$ is

a) $3\frac{1}{4}$ b) $2\frac{1}{4}$ c) $4\frac{1}{3}$

d) $1\frac{3}{4}$ e) None of the above

Solution: c) $4\,^1/_3$

$(1*2) = 2 + \frac{1}{1} = 3$. $(1*2)*4 = 3*4 = 4 + \frac{1}{3} = 4\,^1/_3$.

10) If $b^2 + 7b = 120$, then one possible value for $b^2 - 4b$ is

a) 125 b) 165 c) 15 d) 8 e) 32

Solution: e) 32

$b^2 + 7b - 120 = 0$

$(b - 8)(b + 15) = 0$

So b must be either 8 or -15. $(-15)^2 - 4\cdot(-15) = 285$, which is not an option. $8^2 - 4\cdot 8 = 32$.

11) Carry out the division and simplify $\frac{x^2-4}{x^2+2x-3} \div \frac{x^2-6x+8}{x^2-x-12}$

a) $\frac{x^2-4}{x^2+2x-3}$ b) $\frac{(x+2)(x-2)^2}{(x-1)(x+3)^2}$ c) $\frac{x-4}{x-1}$

d) $\frac{x+2}{x-1}$ e) None of the above

Solution: d) $\frac{x+2}{x-1}$

$$\frac{x^2-4}{x^2+2x-3} \div \frac{x^2-6x+8}{x^2-x-12} = \frac{x^2-4}{x^2+2x-3} \cdot \frac{x^2-x-12}{x^2-6x+8} = \frac{(x+2)(x-2)}{(x+3)(x-1)} \cdot \frac{(x+3)(x-4)}{(x-2)(x-4)} = \frac{x+2}{x-1}$$

12) A store advertised 20% off fashion watches. One watch had a sale price of \$ 36.76. What was the regular price?

a) \$44.11 b) \$29.95 c) \$45.95 d) \$67.17 e) None of the above

Solution: c) \$45.95

Let x be the original price. $x - .2x = 36.76$. Solving for x gives $x = 45.95$.

13) An operator assisted telephone call to another state costs \$10.15 for 15 minutes. The same call dialed directly costs \$0.60 for the first minute and \$0.40 for each additional minute. How much money is saved on a 15 minute call by dialing direct?

a) \$1.15 b) \$2.55 c) \$3.95 d) \$4.55 e) None of the above

Solution: c) \$3.95.
Cost for assisted call for 15 minutes is \$10.15.
The cost for the direct call for 15 minutes is $0.6 + 0.4 \times 14 = 6.2$ (dollars)
The saving is $10.15 - 6.2 = 3.95$ (dollars)

14) How long will it take a carpenter to cut a 4 in by 4 in by 8 ft piece of treated lumber into four equal pieces if each cut takes 2½ minutes?

a) 5 min b) 7.5 min c) 10 min d) 12.5 min e) None of the above

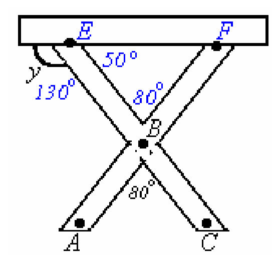
Solution: b) 7.5 min.
It needs three cuts to get four pieces. Each cut takes 2.5 minutes. So, three cuts take $2.5 \times 3 = 7.5$ minutes.

15) A health club has an initiation fee plus a monthly membership fee. Amy paid a total of \$390 for six months of membership plus the initiation fee. Juan paid a total of \$435 for nine months of membership plus the initiation fee. What is the initiation fee?

a) \$15 b) \$30 c) \$250 d) \$300 e) None of the above

Solution: d) \$300
Assuming that the initiation fee is I and the month fee is x.
For Amy: $390 = 6x + \mathrm{I}$ (1)
For Juan: $435 = 9x + \mathrm{I}$ (2)
Solving two equations for x and I: $3x = 435-390 = 45$. So, $x = 15$ and $\mathrm{I} = 390 - 6\times 15 = 300$.
The initiation fee is \$300.

16) The legs of a picnic table with the table top form an isosceles triangle as indicated in the picture (the parts of the legs being the equal sides). If the measure of the angle ABC is 80°, what is the measure of the angle y?



a) 50° b) 130° c) 150°

d) 100° e) None of the above

Solution: b) 130°
The $\angle EBF = \angle ABC = 80°$. Since ΔEBF is an isosceles with $EB = FB$, $\angle BEF = 50°$. Now $\angle y + \angle BEF = 180°$ gives $\angle y = 130°$.

17) The expression $\frac{a}{\sqrt[3]{ab^2}}$ is equivalent to

a) $\frac{\sqrt[3]{ab^2}}{b}$ b) $\sqrt[3]{ab}$ c) $a\sqrt[3]{ab^2}$

d) $\frac{\sqrt[3]{a^2b}}{b}$ e) None of the above

Solution: d) $\frac{\sqrt[3]{a^2b}}{b}$

$$\frac{a}{\sqrt[3]{ab^2}} = \frac{a}{a^{1/3}b^{2/3}} = \frac{a^{2/3}}{b^{2/3}} = \frac{a^{2/3}b^{1/3}}{b} = \frac{\sqrt[3]{a^2b}}{b}$$

18) What is the smallest prime number that is a factor of the sum of 5^{81} and 3^{29}?

a) 3 b) 5 c) 15 d) 2 e) None of the above

Solution: d) 2
5^{81} and 3^{29} are odd numbers. The sum of two odd numbers is even.
Thus, the smallest prime number that is a factor of the sum is 2.

19) A pack of rats abandoned a sinking ship and colonized a deserted island. In the first year their population tripled. In the second year it fell 6 short of doubling. In the third year it increased by 24. In each of the next two years it was cut in half, so that after 5 years, the population stood at 15. How many rats were in the original colony?

a) 4 b) 7 c) 8 d) 27 e) None of the above

Solution: b) 7
Let P be the original population of rats. After 1 year the population is $3P$, since it tripled. After 2 years it was 6 short of doubling, so the population is $2(3P)-6$. Twenty-four more is $2(3P)-6+24=6P+18$, so this is the population after 3 years. Taking half for the next two years, we have $\frac{1}{2}(6P+18)=3P+9$ after 4 years, then $\frac{1}{2}(3P+9)$ after 5 years.

Therefore, $\frac{1}{2}(3P+9)=15$, or $3P+9=30$. Solving, we have $P=7$.

20) A legally caught fish needs to be both in season and of size no less than the legal size limit. If two-fifths of the fish in the lake are out of season, and three-fourths of the fish that are in season are under the legal size limit, what fraction of the fish in the lake are legal to catch?

a) 9/20 b) 3/10 c) 1/5 d) 3/20 e) None of the above

Solution: d) 3/20

If we let T be the total number of fish in the lake, then, since $\frac{2}{5}T$ = the number of fish out of season, $\frac{3}{5}T$ = the number of fish in season. Now, one-fourth of these are at or above the legal size limit, since three-fourths are under the legal size limit. Therefore, $\frac{1}{4}\cdot\frac{3}{5}T = \frac{3}{20}T$ = the number of fish that are legal to catch.

21) Simplify $\frac{x-y}{x^{-2}-y^{-2}}$

a) $\frac{1}{x+y}$ b) $\frac{xy}{x+y}$ c) $x+y$
d) x^3-y^3 e) None of the above

Solution: e) None of the above.

We can write this as $\frac{x-y}{\frac{1}{x^2}-\frac{1}{y^2}}$. Multiplying through by the least common denominator, x^2y^2, in the numerator and denominator, we have

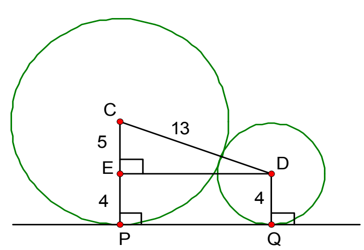
$$\frac{x-y}{\frac{1}{x^2}-\frac{1}{y^2}} = \frac{x^2y^2(x-y)}{x^2y^2\left(\frac{1}{x^2}-\frac{1}{y^2}\right)} = \frac{x^2y^2(x-y)}{y^2-x^2} = \frac{x^2y^2(x-y)}{(y-x)(y+x)} = \frac{-x^2y^2}{x+y}.$$

22) Circles of radius 4 and 9 are tangent to each other and to a horizontal line. Find the distance *d(P,Q)* between the two points of tangency with the line

a) 6 b) $\sqrt{56}$ c) 12
d) $9\sqrt{2}$ e) 13

Solution: c) 12

See the figure**.** If C and D are the centers of the two circles then DC = 13 = 9 + 4. The radii are perpendicular at the tangent points, so if we draw DE parallel to PQ, then DE is perpendicular to PC. Therefore, ΔCED is a right triangle with CE = 5 = 9 – 4. So, by the Pythagorean Theorem, the third side is $\sqrt{13^2-5^2}=12$, which is the same as $d(P,Q)$.



23) The sum of the distinct solutions of the equation $(x-\frac{2}{x})^2-2x+\frac{4}{x}=-1$ is

a) 1 b) -1 c) -3 d) 3 e) None of the above

Solution: a) 1

Rewriting $(x-\frac{2}{x})^2-2x+\frac{4}{x}=-1$, we have $(x-\frac{2}{x})^2-2(x-\frac{2}{x})+1=0$. If we let $u=x-\frac{2}{x}$, then this is the quadratic $u^2-2u+1=0$ or $(u-1)^2=0$. This has solution $u=1$, so $1=x-\frac{2}{x}$. Multiplying by x, we have $x=x^2-2$ or $x^2-x-2=0$. Factoring, then, $(x-2)(x+1)=0$. This has solutions 2 and -1, which sum to 1.

24) The size of a TV screen is described by the length of its diagonal. The ratio of the height to width is the same for all televisions. How much larger is the area of a 30" screen than the area of 20" screen?

a) The areas are the same
b) The ratio of the areas is 3:2
c) The ratio of the areas is 2:1
d) The ratio of the areas is 4:1
e) The ratio of the areas is 9:4

Solution: e) The ratio of the areas is 9:4

Let $h_{30}, w_{30}, A_{30}, h_{20}, w_{20}, A_{20}$ be the height, width, area of the 30 inch and 20 inch televisions, respectively. Now $\frac{h_{30}}{w_{30}}=\frac{h_{20}}{w_{20}}=r$ (for some r) for both, so $h_{30}=rw_{30}$ and $h_{20}=rw_{20}$.

But, $\frac{A_{30}}{A_{20}}=\frac{h_{30}\cdot w_{30}}{h_{20}\cdot w_{20}}=\frac{rw_{30}{}^2}{rw_{20}{}^2}=\frac{w_{30}{}^2}{w_{20}{}^2}$, and by the Pythagorean Theorem

$h_{30}{}^2+w_{30}{}^2=(rw_{30})^2+w_{30}{}^2=w_{30}{}^2(r^2+1)=30^2$, so $w_{30}{}^2=\frac{900}{r^2+1}$. Similarly,

$w_{20}{}^2=\frac{400}{r^2+1}$. Therefore, $\frac{A_{30}}{A_{20}}=\frac{w_{30}{}^2}{w_{20}{}^2}=\frac{900}{400}=\frac{9}{4}$.

[*Alternative solution:* The fact that the ratio of the height to width is the same for all TV sets, makes all screens geometrically similar. The linear scaling factor from the 20'' to the 30'' is 3/2. The scaling factor for areas of geometrically similar shapes is the square of the linear scaling factor, hence in our situation, the area is scaled by a factor of 9/4.]

25) Which of the following numbers are prime?

a) 417 b) 847 c) 419 d) 143 e) 847 and 419 are both prime

Solution: c) 419

Checking each number: The digits of 417 sum to 4 + 1 + 7 = 12, which is divisible by 3. So, 417 is divisible by 3. Now $847=7\cdot 121$ and $143=13\cdot 11$, so the only prime is 419.