Finite Geometry Exercises

Using the following axiomatic system:

**Axiom 1:** There exist exactly five points.

**Axiom 2:** Any two distinct points have exactly one line on both of them.

**Axiom 3:** Each line has exactly two points.

1) Construct a model of this finite geometry.
2) How many lines are there in this geometry? Prove your answer.
3) Given a line and a point not on the line, how many lines are there containing the given point that are parallel to the given line? If two lines are parallel to the same line, are they necessarily parallel to each other? Give examples from your model – no proofs necessary here.
4) Prove that not all points are on the same line.
5) Prove that each point has exactly four lines on it.