4. Area is determined by the definite integral \( A = \int_2^3 (x^2 - 2x + 1) \, dx \).

\[ f(x) = x^2 - 2x + 1 \quad \text{and} \quad F(x) = \frac{x^3}{3} - 2\left(\frac{x^2}{2}\right) + x = \frac{x^3}{3} - x^2 + x \]

\( F(x) \) is an antiderivative of \( f(x) \). This can be verified by seeing that \( F'(x) = f(x) \).

Now

\[
\int_2^3 (x^2 - 2x + 1) \, dx = F(3) - F(2) = \left(\frac{3^3}{3} - 3^2 + 3\right) - \left(\frac{2^3}{3} - 2^2 + 2\right) = (9 - 9 + 3) - \left(\frac{8}{3} - 4 + 2\right) = 3 + 4 - 2 - \frac{8}{3} = \frac{7}{3} = A
\]

5. \( A = \int_{-2}^{2} x^2 \, dx = F(2) - F(-2) = \frac{2^3}{3} - \left(\frac{(-2)^3}{3}\right) = \frac{8}{3} - \left(-\frac{8}{3}\right) = \frac{16}{3} \quad \text{where} \ f(x) = x^2 \ , \ F(x) = \frac{x^3}{3} \)

6. a) \( A = \int_0^2 x^3 \, dx = F(2) - F(0) = \frac{2^4}{4} - \frac{0^4}{4} = \frac{16}{4} - 0 = 4 \quad \text{where} \ f(x) = x^3 \text{and} \ F(x) = \frac{x^4}{4} \)

\[
\int_{-2}^{2} x^3 \, dx = F(2) - F(-2) = \frac{2^4}{4} - \left(-\frac{2}{4}\right)^4 = \frac{16}{4} - \frac{16}{4} = 0
\]

b) This is not an area. Examine the graph of \( y = x^3 \) on the interval from \( x=-2 \) to \( x=2 \). The two pieces cancel one another to give a result of 0.

7. \( A = \int_0^b \left(\frac{h}{b}\right) x \, dx = F(b) - F(0) = \frac{1}{2b} \cdot \frac{h b^2}{b} = \frac{h b}{2} - 0 = \frac{1}{2} b h \). Note this is the formula for the area of a triangle. (Here \( f(x) = \left(\frac{h}{b}\right) x \) and \( F(x) = \left(\frac{h}{b}\right) \frac{x^2}{2} = \frac{h x^2}{2b} \))

8. a) \( \int_1^2 \frac{1}{x} \, dx = F(2) - F(1) = \ln 2 - \ln 1 = \ln 2 - 0 = \ln 2 \approx 0.693 \). Here \( f(x) = \frac{1}{x} \) and \( F(x) = \ln x \).

b) \( \int_1^e \frac{1}{x} \, dx = \ln e - \ln 1 = \ln e - 0 = 1 \)