

Name _____ points of 153
_____ %

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (15) Determine whether or not the following argument is valid. (First write the argument form symbolically.)

If I work all night on this homework, then I can answer all the exercises. Either I can't answer all the exercises or I understand the material. It's not true that I understand the material and do not pass the course. I worked all night on this homework. Therefore I will pass the course.

2. (15) Give a proof by contradiction of "The circle whose equation is $x^2 + y^2 = 2$ and the line whose equation is $y = x + 4$ do not intersect."

3. (10) Use mathematical induction to prove that 3 divides $n^3 + 2n$ whenever n is a non-negative integer.

4. (8) The population on New York City is 7,000,000. If a person has no more than 500,000 hairs on his head, what is the minimum number of people that have the same number of hairs on their heads?

5. (11) There are 7 math books, 7 physics books, and 7 CS books that are to be placed on a shelf.

a. (3) How many ways can this be done if the books must alternate by subject in this way: 1st math, 2nd physics, 3rd CS, 4th math, 5th physics, etc.?

b. (4) How many ways can this be done if the books must alternate by subject with no restriction on the order of the subjects, but with the same sequence throughout?

c. (4) How many ways can you choose 3 math books, 2 physics books, and 5 CS books to give to a friend?

6. (12) a. A candy store sells 6 kinds of chocolates. How many ways are there to fill a box with chocolates if the box holds 10 pieces?

b. How many ways if the box is to hold at least one piece of each kind?

7. (10) How many ways are there to place 100 people in 3 different rooms, if each room is to be occupied and it does not matter which people go into which room?

8. (10) In a survey of 270 college students, it is found that 64 like brussels sprouts, 94 like broccoli, 58 like cauliflower, 26 like both brussels sprouts and broccoli, 28 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower, and 14 like all three vegetables. How many of the 270 students do not like any of these vegetables?

9. (20) A market sells 10 kinds of soda. You want to buy 12 bottles. How many possibilities are there if you want

a. (5) at least of one of each kind?

b. (15) at most 3 bottles of any kind?

10. (12) An office manager has 4 employees and 4 reports to be done. In how many ways can the reports be assigned if each employee has at least one report to do?

11. (10) You take a job that pays \$25,000 per year. Each year you receive a raise of \$500 plus 2% of the previous year's salary.. Set up a recurrence relation (including initial conditions) for your salary after n years. *Do not solve the recurrence relation.*

12. (10) Solve the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$, $a_0 = 1$, $a_1 = 5$.

13. (10) Modify the problem of #12 thus: $a_n = 3a_{n-1} + 4a_{n-2} + n$, $a_0 = 1$, $a_1 = 5$. Now solve.

