MATH 300 (2028) Test 1 2001 Feb 21

Name________________________________ ___points of 148

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Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

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1. (20) On one tableau prepare truth tables for:

   a. $\neg q \rightarrow p$ b. $(p \vee q) \wedge [\neg(p \wedge q]$ c. $\neg(\neg p \wedge \neg q)$ d. $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$

2 (6) Of the propositions in problem #1 which

   a. are equivalent?

   b. are contradictions?

   c. are tautologies?

3. (5) Give the inverse of the converse of "If I have a dollar, then I shall buy a beer."

4. (5) Write in symbolic form: "Ron likes this problem, unless this problem cannot be solved." Use only positive (un-negated) simple propositions.

5. (9) a. Write symbolically using quantifiers: "Someone loves everyone."

   b. Form the symbolic negation of #a.

   c. Write the negation of #a in plain English.

6. (10) Using logic, prove $A \subseteq B \rightarrow \overline{B} \subseteq \overline{A}$.

7. (5) Draw a Venn diagram for $A \subseteq B \cap C$.

8. (10) Prove: Let $f: A \rightarrow B$ and sets S and T $\subseteq B$. Then $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.

9. (10) a. Write the floor (greatest integer) function $\lfloor x \rfloor$ as a piecewise-defined function.

b. Write $\lfloor \frac{1}{2} x \rfloor$ as a piecewise-defined function.

10. (10) Prove: If A is a countable set and B is a countable set, then $A \cup B$ is countable.

11. (10) $f(n) = (x^3 + x^2 \log(x))(\log(x) + 1)$ us $O(x^n)$ for what least integer n? Prove your result.

12. (10) a. Write $(101011)_2$ in base 10 expansion.

b. Wrote 67_{10} is base 2 expansion.

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13. (10) Use the Euclidean algorithm to find the gcd(272,1479) and express this gcd as a linear combination of 272 and 1479: gcd = 272s + 1479t.

14. (10) a. Find an inverse of 9 modulo 26.

b. Solve $9x-5 \equiv 2 \pmod{26}$

15. (8) Find the non-negative number a less than 20 represented by $(2,4) \equiv (a \bmod 4,\ a \bmod 5)$.

16. (10) Compute:

a. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} =$

b. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \odot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} =$

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