MATH 300 (2028) Test 1 2002 Feb 12

Name ANSWER KEY ___points of 111 ______%

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (20) On one tableau prepare truth tables for:

a. $\neg p \vee q$ b. $\neg(p \wedge \neg q) \leftrightarrow (p \rightarrow q)$ c. $[p \wedge (p \rightarrow q)] \rightarrow q$ d. $[(p \wedge q) \rightarrow p] \rightarrow p$

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$p \rightarrow q$	$\neg(p \wedge \neg q) \leftrightarrow (p \rightarrow q)$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$	$p \wedge q$	$(p \wedge q) \rightarrow p$	$[(p \wedge q) \rightarrow p] \rightarrow p$
T	T	F	F	T	F	T	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T	F	T	F	T	T
F	T	T	F	T	F	T	T	T	F	T	F	T	F
F	F	T	T	T	F	T	T	T	F	T	F	T	F
				a				b		c			d

2 (6) Of the propositions in problem #1 which

a. are equivalent? $\boxed{b \Leftrightarrow c}$

b. are contradictions? $\boxed{\text{None}}$

c. are tautologies? $\boxed{b, c}$

3. (6) Give the contrapositive of the converse of "If it rains, then it pours."

$r \equiv$ it rains; $p \equiv$ it pours

Given: $r \rightarrow p$

Converse of given: $p \rightarrow r$

Contrapositive of converse: $\neg r \rightarrow \neg p$

$\boxed{\text{If it does not rain, then it does not pour.}}$

4. (10) Use proved theorems to simplify $\neg p \wedge (q \vee p)$

$(\neg p \wedge q) \vee (\neg p \wedge p)$ Distributive law

$(\neg p \wedge q) \vee F$ (Equivalence from Table 6)

$\neg p \wedge q$ (Identity law)

5. (9) a. Write symbolically using quantifiers: "Everyone has someone they love."

$xLy \equiv x$ loves y $\boxed{\forall x \, \exists y \, (xLy)}$

b. Form the symbolic negation of #a.

$\neg[\forall x \, \exists y \, (xLy)] \equiv \exists x \, \neg \exists y \, (xLy) \equiv \boxed{\exists x \, \forall y \, (\neg xLy)}$

c. Write the negation of #a in plain English.

$\boxed{\text{There is someone who loves no one.}}$

6. (10) Using logic, prove $A \cap B \subseteq A$.

$x \in A \cap B \Leftrightarrow x \in A \wedge x \in B \Rightarrow x \in A$ $(p \wedge q \rightarrow p)$

$\therefore A \cap B \subseteq A$

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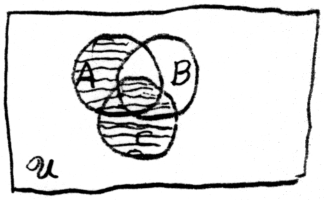
7. (5) Draw a Venn diagram for (A\B)∪C. ≡ (A−B)∪C



8. (10) Prove: Let f:A→B and sets S and T ⊆B. Then $f^{-1}(S\cup T) = f^{-1}(S)\cup f^{-1}(T)$.

$x \in f^{-1}(S\cup T) \Leftrightarrow f(x) \in S\cup T \Leftrightarrow f(x) \in S \vee f(x) \in T$

$\Leftrightarrow x \in f^{-1}(S) \vee x \in f^{-1}(T) \Leftrightarrow x \in f^{-1}(S)\cup f^{-1}(T)$

$\therefore f^{-1}(S\cup T) = f^{-1}(S)\cup f^{-1}(T)$

9. (10) a. *Prove* that f: $\mathfrak{R}\to\mathfrak{R}$ given by f(x) = 2x+1 is one-to-one (injective).

Assume that $f(a)=f(b)$.

$2a+1 = 2b+1$

$2a = 2b$

$a = b$ $\quad \therefore$ f is injective

b. Find the function inverse to f.

Let $y = 2x+1$

$y-1 = 2x$

$x = \frac{1}{2}(y-1)$

$\boxed{f^{-1}(x) = \frac{1}{2}(x-1)}$

Check: $f\circ f^{-1}(x) = 2[\frac{1}{2}(x-1)]+1 = x-1+1 = x$

$f^{-1}\circ f(x) = \frac{1}{2}[(2x+1)-1] = \frac{1}{2}[2x] = x$

10. (7) Prove: The set of fractions of the form $\frac{1}{n}$ is countable.

The mapping $n \longleftrightarrow \frac{1}{n}$ is a bijection.
Therefore the set of integers and the set of fractions of the form $\frac{1}{n}$ are equinumerous.

11. (10) $f(x) = 2x^2+x^3\log x$ is $O(x^n)$ for what least integer n? Prove your result.

$\log x$ is of order $O(x^{1/n})$ for any n. Hence $x^3\log x$ is of $O(x^3)$.
$2x^2$ is of $O(x^2)$. $\therefore 2x^2+x^3\log x$ is of $\boxed{O(x^3)}$

12. (8) What is the cardinality of each of the following sets?

a. ∅ 0

b. {∅} 1

c. {∅, {∅}} 2

d. {∅, {∅}, {∅, {∅}}} 3

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