Supplement to Test 2

1. (32) For the polynomial $p(x) = 2x^5+x^4-6x^3-3x^2-8x-4$, $p(-x) = -2x^5+x^4+6x^3-3x^2+8x-4$

1 change — 4 changes

a. (6) Use Descartes' rule of signs and other facts to give the possibilities for the roots: positive, negative, complex.

deg	5	5	5
pos	1	1	1
neg	4	2	0
cx	0	2	4

b. (6) Use synthetic division to find an upper bound & a lower bound for the roots.

	2	1	−6	−3	−8	−4	
1	2	3	−3	−6	−11	−15	
2	2	5	4	5	2	0	Upper bound & only positive zero root
−2	2	−1	2	−1	0	=	
−3	2	−5	17	−50	=	≠	Lower bound.

Notice that whenever a root has been found (on lines 3 & 4), we continue testing with the reduced polynomial (i.e. the quotient.)

Our knowledge as of now: $p(x) = (x-2)(x+2)(2x^3-5x^2+17x-50)$

c. (5) Use the rational roots theorem to list all possible rational roots.

$r = c/d$: c divides −4 so $c = \pm1, \pm2, \pm4$; d divides 2 so $d = \pm1, \pm2$.

$\therefore r = \pm1, \pm\frac{1}{2}, \pm2, \pm4$

d. (5) Pare down the list of possible rational roots.

±4 out of bounds; $\pm2, 1$ already checked; $\frac{1}{2}$ excluded because only 1 root >0, and that is 2. $\boxed{-1, -\frac{1}{2}}$

e. (10) Find all roots of the polynomial. $\boxed{r = 2, -2, -\frac{1}{2}, \pm i}$

	2	1	2	1	
−1	2	−1	3	−2	
−½	2	0	2	0	root

This table used the coefficients from the last quotient in the preceding table.

Now, $p(x) = (x-2)(x+2)(x+\frac{1}{2})(2x^2+2)$

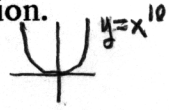
$= 2(x-2)(x+2)(x+\frac{1}{2})(x^2+1)$

$\therefore x^2+1=0 \Rightarrow x^2=-1 \Rightarrow x=\pm i$

2. (20) For the polynomial $p(x) = (x-1)(x+1)^2(x-2)^3(x+2)^4$,

a. (3) Approximate p(x) by a power function.

$p(x) \approx x \cdot x^2 \cdot x^3 \cdot x^4 = \boxed{x^{10}}$



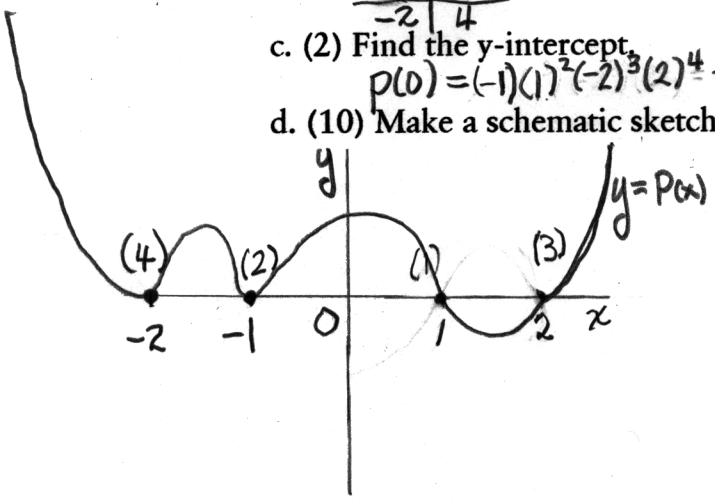
b. (5) List the zeroes and their multiplicities.

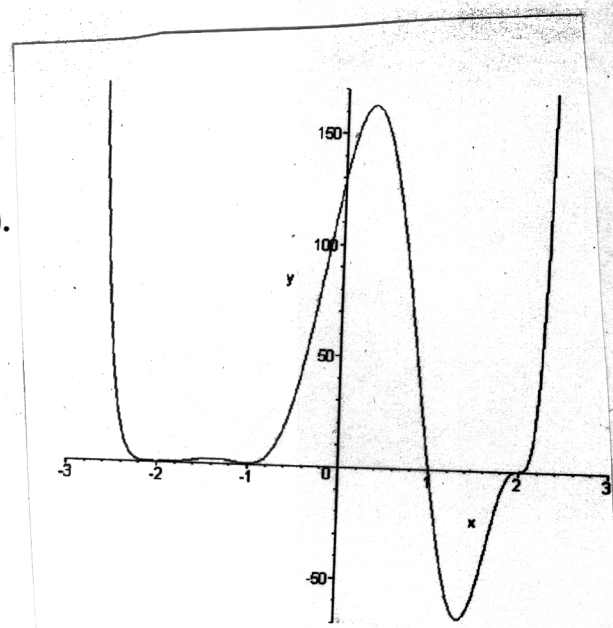
Zero	Mult
1	1
-1	2
2	3
-2	4

c. (2) Find the y-intercept.

$p(0) = (-1)(1)^2(-2)^3(2)^4 = -(-8)(16) = \boxed{128}$

d. (10) Make a schematic sketch of the graph of p(x).

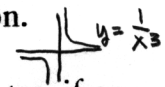




3. (28) For the rational function $r(x) = \dfrac{(x+1)(x-2)^2}{x(x-1)^2(x+2)^3}$,

a. (3) Approximate r by a power function.

$r \approx \dfrac{x \cdot x^2}{x \cdot x^2 \cdot x^3} = \dfrac{x^3}{x^6} = \boxed{\dfrac{1}{x^3}}$



b. (3) Determine the horizontal asymptotes, if any.

$\boxed{y = 0}$

c. (2) Find the y-intercept, if any.

No y-intercept

c. (5) List the zeroes and their multiplicities.

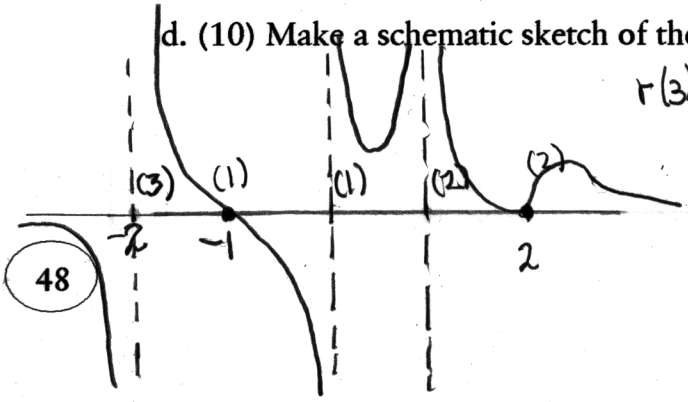
d. (5) List the vertical asymptotes and their multiplicities.

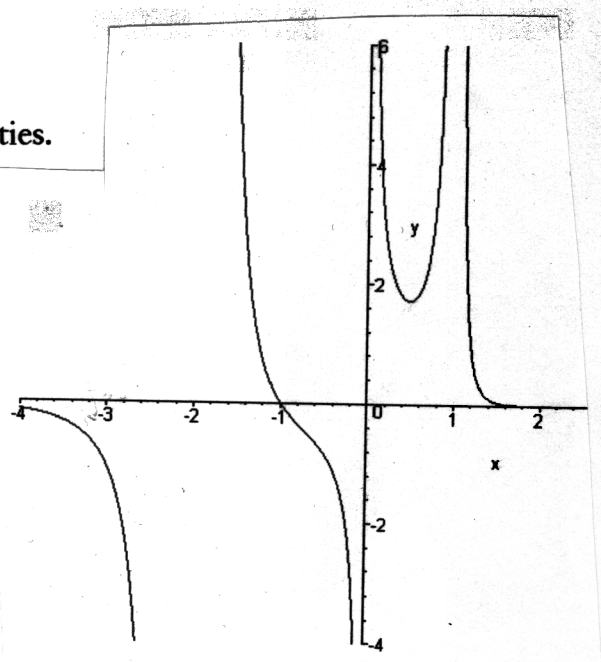
Zero	mult
-1	1
2	2

VA	mult.
0	1
1	2
-2	3

d. (10) Make a schematic sketch of the graph of r(x).

$r(3) = \dfrac{4 \cdot 1^2}{3 \cdot 2^2 \cdot 5^3} > 0$





4. (7) Find a polynomial p of least degree that has as roots 0, 1, -5, and 5 of multiplicities 1, 2, 1, and 1 respectively, and which satisfies $p(-1) = 6$.

$p(x) = cx(x-1)^2(x+5)(x-5)$

$6 = p(-1) = c(-1)(-2)^2(4)(-6) = 96 \Rightarrow c = \frac{1}{16}$

$\boxed{p(x) = \frac{1}{16}x(x-1)^2(x+5)(x-5)}$