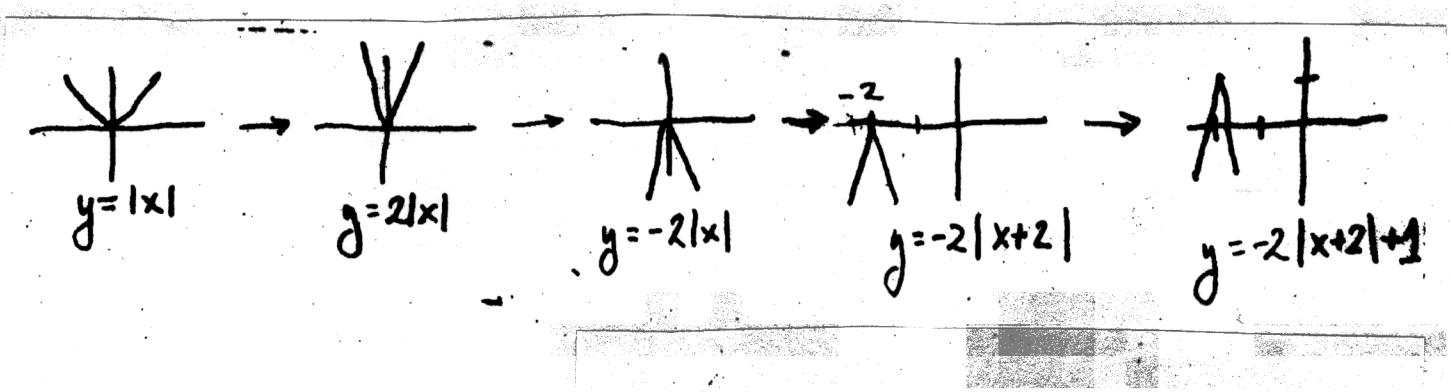
MATH 120A **Test 2** 2011 April 18

Name ANSWER KEY ____ points of ~~144~~ 139 ____%

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (15) In the space below make a rough sketch of $f(x)=-2|x+2|+1$ by showing the graph's transformation from a basic shape by a sequence of steps.



2. (10) For the circle $x^2+y^2-6x+4y=36$ $\Leftrightarrow (x^2-6x)+(y^2+4y)=36 \Leftrightarrow (x^2-6x+9)+(y^2+4y+4)$
$\Leftrightarrow (y-3)^2+(y+2)^2=49=7^2$ $=36+9+4=49$

a. center = $(3,-2)$

b. radius = 7

3. (~~24~~ 19) For the quadratic function $f(x)=-x^2+6x-8$

a. (1) does the parabola open up or down? Down

b. (3) vertex (both coordinates) = $(3,1)$ $x=-\frac{b}{2a}=-\frac{6}{2(-1)}=3$, $y=f(3)=-3^2+6\cdot 3-8=1$

c. (6) x-intercepts = $2, 4$

$-x^2+6x+8=0 \Leftrightarrow x^2-6x+8=0 \Leftrightarrow (x-2)(x-4)=0$

d. (3) axis: $x=3$

e. (3) range= $(-\infty, 1]$

f. (3) y-intercept= $-8 = f(0)$

44 ~~49~~

4. (6) For the straight line $-2x+7y=17 \Leftrightarrow 7y = 2x+17 \Leftrightarrow y = \frac{2}{7}x + \frac{17}{7} = \frac{2}{7}\left(x + \frac{17}{2}\right)$

a. slope = $\boxed{\frac{2}{7}}$

b. y-intercept = $\boxed{\frac{17}{7}}$

c. x-intercept = $\boxed{-\frac{17}{2}}$

5. (10) For the line £ of #4 and the point P:(-2,3), give the equation of the line through P

a. parallel to £: $\boxed{y-3=\frac{2}{7}(x+2)}$ $y = \frac{2}{7}x + \frac{25}{7}$

b. perpendicular to £: $\boxed{y-3=-\frac{7}{2}(x+2)}$ $y = -\frac{7}{2}x - 4$

6. (16) For $f(x)=x^2+1$ and $g(x)=\sqrt{x}$, find

a. (2) $g \circ f(x) = g(f(x)) = g(x^2+1) = \boxed{\sqrt{x^2+1}}$

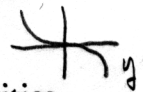
b. (2) $f \circ g(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2+1 = \boxed{x+1}$

c. (4) domain of $f \circ g = \boxed{[0,\infty)}$

d. (8) $\frac{f(2+h)-f(2)}{h} = \frac{[(2+h)^2+1]-[2^2+1]}{h} = \frac{4+4h+h^2+1-4-1}{h} = \frac{h(4+h)}{h} = \boxed{4+h}$

7. (22) For the polynomial $p(x) = -2x^2(x+1)(x-1)^2(x+2)^3(x-2)$

a. (5) Approximate p(x) by a power function.
$p(x) \approx -2x^2 \cdot x \cdot x^2 \cdot x^3 \cdot x = \boxed{-2x^9}$

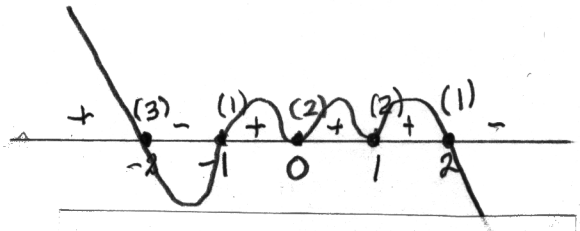


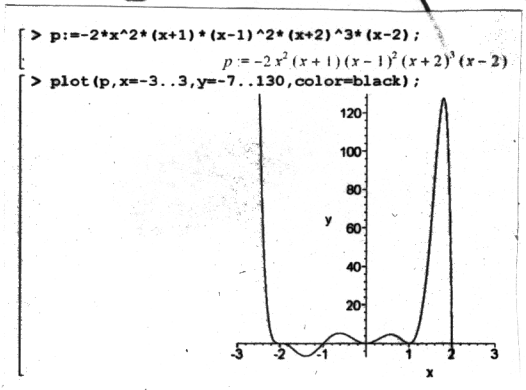
b. (5) Find the zeroes of p(x) and their multiplitities.

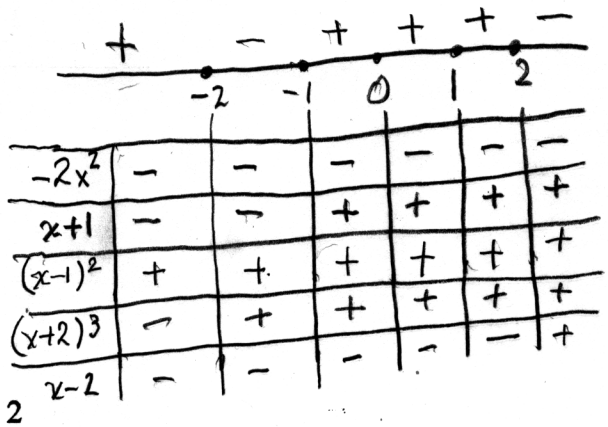
zero	multiplicity
0	2
-1	1
1	2
-2	3
2	1

c. (2) Find the y-intercept of p(x).
$\boxed{p(0)=0}$

d. (10) Plot the graph of p(x) below.







8. (20) A cable TV firm presently serves 5000 households and charges $20 per month. For each decrease of $1 in the monthly rate 500 new customers will subscribe. Let R(x) = monthly revenue when the monthly charge is x dollars.

a. Determine the revenue function R(x) = $(15000-500x)x = -500x^2+15000x$

R = no. of customers × monthly rate

Let x = monthly rate. Then no. of customers = $5000+(20-x)500$

$= 15000-500x$

b. Find the value of x that results in the maximum monthly revenue.

R(x) is maximized when $x = \frac{-b}{2a} = \frac{-15000}{2(-500)} =$ **$15 per month**

9. (21) For the polynomial $p(x) = x^4-7x^3+14x^2-3x-9$, 3 changes

$p(-x) = x^4+7x^3+14x^2+3x-9$ 1 change

a. (4) Use Descartes' rule of signs and other facts to give the possibilities for the roots: positive, negative, zero.

deg	4	4
pos	3	1
neg	1	1
0	0	2

b. (4) Use synthetic division to find an upper bound & a lower bound for the roots.

7 | 1 -7 14 -3 -9
7 0 98 665
1 0 14 95 656

∴ 7 is an upper bound

-1 | 1 -7 14 -3 -9
-1 8 -22 25
1 -8 22 -25 16

∴ -1 is a lower bound

c. (4) Use the rational roots theorem to list all possible rational roots.

$\frac{c}{d} = \frac{\text{factor of } 9}{\text{factor of } 1} = \frac{\pm1,\pm3,\pm9}{\pm1} =$ **$\pm1, \pm3, \pm9$**

d. (3) Pare down the list of all possible rational roots using bounds & previous divisions.

1, 3

e. (6) Find all roots of the polynomial.

1 | 1 -7 14 -3 -9
1 -6 8 5
1 -6 8 5 -4

3 | 1 -7 14 -3 -9
3 -12 6 9
3 | 1 -4 2 3 0
3 -3 -3
1 -1 -1 0

∴ 3 is a root

∴ 3 is (double) root

3

$x^2-x-1=0 \Rightarrow x = \frac{1\pm\sqrt{(-1)^2-4\cdot1(-1)}}{2\cdot1} = \frac{1\pm\sqrt{5}}{2}$

3, 3, $\frac{1\pm\sqrt{5}}{2}$