

MATH 120A

Mathematical Recreations

1. Prove the following result due to Cardano: There are not two numbers that sum to 10 and whose product is 40. [Hint: quadratic maximization.]

2. Consider the function $f(x) = \frac{4x+3}{x^6+1} + 2^x - 2$. What *function* does $f(x)$ approach as $x \rightarrow \pm\infty$? What *value* does $f(x)$ approach as $x \rightarrow \infty$? What *value* does $f(x)$ approach as $x \rightarrow -\infty$? (from William Dunham's *The Mathematical Universe*.)

3. A lightbulb upstairs is controlled by one of three on-off switches on the first floor. The bulb is known to be off at the present time. How would you determine which switch controlled the bulb, if you could manipulate the switches in any way, but were allowed only one trip upstairs?