MATH 120A (2506) Test 2 2005 March 28

Name ANSWER KEY ___points of 149 ______%

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (32) For the polynomial $p(x) = 2x^5+5x^4-8x^3-14x^2+6x+9$, 2 changes

$p(-x) = -2x^5+5x^4+8x^3-14x^2-6x+9$ 3 changes

a. (6) Use Descartes' rule of signs and other facts to give the possibilities for the roots: positive, negative, complex.

degree	5	5	5	5
positive	2	2	0	0
negative	3	1	3	1
complex	0	2	2	4

b. (6) Use synthetic division to find an upper bound & a lower bound for the roots.

x	2	5	-8	-14	6	9	
2	2	9	10	6	18	45	$\boxed{\text{upper bound} = 2}$
-3	2	-1	-5	1	3	0	root = -3
-4	2	-9	31	-123	493		$\boxed{\text{lower bound} = -4}$
1	2	1	-4	-3	0		root = 1
-1	2	-1	-3	0			root = -1

c. (5) Use the rational roots theorem to list all possible rational roots.

factors of $c = 9$: $\pm1, \pm9, \pm3$

factors of $d = 2$: $\pm1, \pm2$

$r = \frac{c}{d} = \boxed{\pm1, \pm9, \pm3, \pm\frac{1}{2}, \pm\frac{9}{2}, \pm\frac{3}{2}}$

d. (5) Pare down the list of possible rational roots.

$\boxed{\pm1, \pm\frac{1}{2}, \pm\frac{3}{2}}$

e. (10) Find all roots of the polynomial.

By factor theorem $p(x) = (x+3)(x-1)(x+1)(2x^2-x-3)$

$2x^2-x-3=0$ $2(-3)=-6=(-3)(2)$

$2x^2-3x+2x-3=0$

$x(2x-3)+(2x-3)=0$

$(x+1)(2x-3)=0$

$x=-1, \frac{3}{2}$

$\therefore$ Roots are: $\boxed{-3, 1, -1, -1, 3/2}$

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2. (28) For the rational function $r(x) = \dfrac{(2x-1)(x+4)^2}{(x-1)(x+2)^2}$,

a. (3) Approximate r by a power function. $r \approx \dfrac{2x \cdot x^2}{x \cdot x^2} = \boxed{2}$

b. (3) Determine the horizontal asymptotes, if any. $\boxed{y=2}$

c. (2) Find the y-intercept, if any. $r(0) = \dfrac{(-1)\,4^2}{(-1)\,2^2} = \boxed{4}$

c. (5) List the zeroes and their multiplicities.

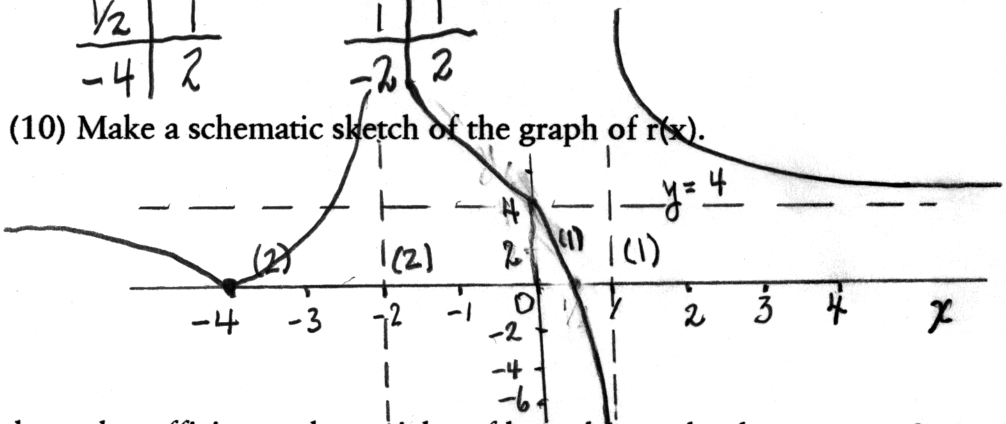
d. (5) List the vertical asymptotes and their multiplicities.

Zero	Multip.
1/2	1
-4	2

VA	Multp.
1	1
-2	2

Vertical asym – 4 pts
Horizontal asym – 2 pts
Zeroes – 4 pts.

d. (10) Make a schematic sketch of the graph of r(x).



3. (7) Find a real-coefficient polynomial p of least degree that has as roots 3, 4, -1 and i of multiplicities 1, 2, 1, and 1 respectively, and which satisfies p(1) = -3.

$p(x) = c(x-3)(x-4)^2(x+1)(x-i)(x+i)$
$= c(x-3)(x-4)^2(x+1)(x^2+1)$
$-3 = p(1) = c(-2)(-3)^2(2)(2) = -72c \Rightarrow c = \frac{-3}{-72} = \frac{1}{24}$

$\boxed{p(x) = \frac{1}{24}(x-3)(x-4)^2(x+1)(x^2+1)}$

Linear factors – 3 pts
Irreducible quadratic – 2 pts
Constant – 2 pts

4. (6) Find the center & radius of the circle whose equation is $2x^2+2y^2-10x+16y+21=0$.

$(2x^2-10x) + (2y^2+16y) = -21$
$2(x^2-5x) + 2(y^2+8y) = -21$
$2(x^2-5x+\frac{25}{4}) + 2(y^2+8y+16) = -21 + \frac{25}{2} + 32$
$2[(x-5/2)^2 + (y+4)^2] = \frac{47}{2}$

$(x-5/2)^2 + (y+4)^2 = 47/4$

$\boxed{C = (5/2, -4) \quad r = \frac{\sqrt{47}}{2}}$

5. (10) The heat experienced by a camper at a campfire is directly proportional to the amount of wood on the fire and inversely proportional to the cube of the distance from the fire.

a. (4) Find a formula for the heat H in terms of the amount of wood W, the distance D ~~of the camper~~ from the fire, and a coefficient of proportionality k.

$\boxed{H = k\frac{W}{D^3}}$

b. (6) If the camper is 20 ft from the fire, and someone doubles the amount of wood, how far from the fire would he need to be so that he feels the same heat as before?

Before wood added: $H = \frac{kW}{20^3}$,
After wood added: $H = \frac{k(2W)}{D^3}$
$\Rightarrow \frac{kW}{20^3} = \frac{2kW}{D^3} \Rightarrow D^3(kW) = (kW)\cdot 2\cdot 20^3 \Rightarrow D^3 = 2\cdot 20^3$

$\Rightarrow \boxed{D = 20\sqrt[3]{2} \approx 25.198}$

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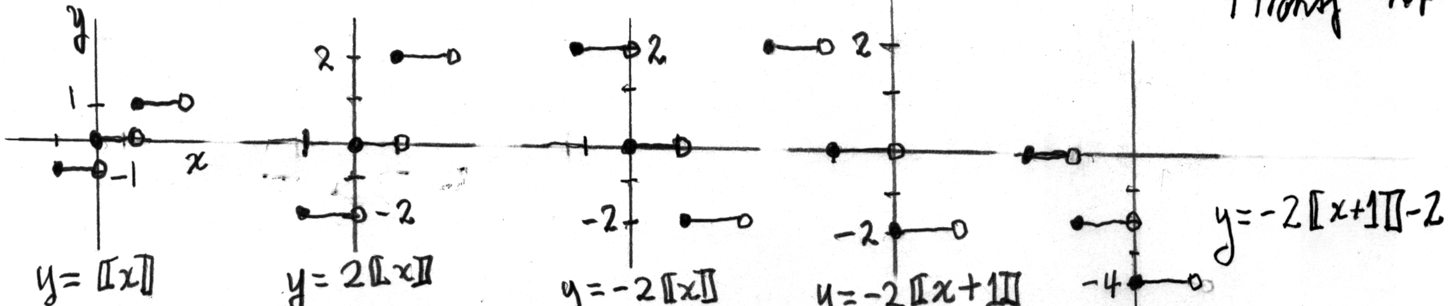
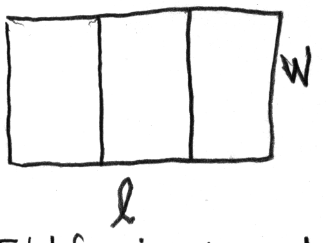
6. (20) Obtain the graph of f(x) = -2 [x+1] -2 from a basic shape by a sequence of transformations.

Basic shape-4 p
4 transf - 16 pts



7. (20) A farmer wants to install a fence around a rectangular field and then divide the field into three rectangular plots by placing two fences parallel to one of the sides. If the farmer can afford only 1000 yards of fencing, which dimensions will give the maximum rectangular area?



Total fencing = 1000 yd

To maximize area $A = lw$

$1000 = \text{Total fencing} = 2l + 4w \Rightarrow 500 = l + 2w$

$\Rightarrow l = 500 - 2w$

To maximize $A = (500-2w)w = -2w^2 + 500w$

Maximum occurs at $w = -\frac{b}{2a} = -\frac{500}{2(-2)} = 125$

$\Rightarrow l = 500 - 2\cdot 125 = 250$

$\boxed{\text{Maximum area of } 31,250\,yd^2 \text{ when } w = 125 \text{ \& } l = 250}$

8. (6) For the line -2x-3y = 6, find $\Rightarrow -3y = 2x + 6 \Rightarrow y = -\frac{2}{3}x - 2$

slope = $\boxed{-2/3}$

y-intercept = $\boxed{-2}$

x-intercept = $\boxed{-3}$ $\quad -2x - 3\cdot 0 = 6 \Rightarrow x = -3$

9. (5) Find the equation of the line through (-2, 5) that is perpendicular to the line of #8.

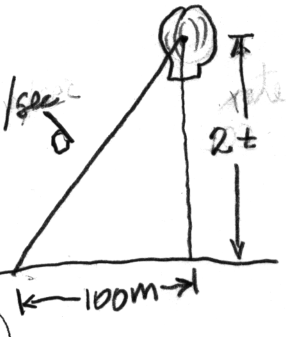
Slope of line sought $= 3/2$. $\therefore$ by point-slope form $\boxed{y - 5 = 3/2(x+2)}$

$y = 3/2x + 8$

$3x - 2y = -16$

10. (15) A hot-air balloon is released at 1:00 p.m. and rises vertically at a rate of 2 m/sec. An observation point is situated 100 meters from a point on the ground directly below the balloon. If t denotes the time (in sec) after 1:00 p.m., express the distance d between the balloon and the observation point as a function of t.

rate of rise = 2m/sec



Distance = Rate × Time

$\therefore$ height of balloon at time $t = 2t$

By the Pythagorean thm,

$d^2 = 100^2 + (2t)^2$

$\therefore d = \sqrt{100^2 + 4t^2} = \boxed{2\sqrt{50^2 + t^2}}$

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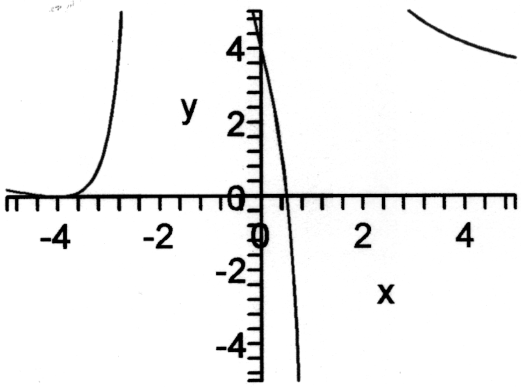
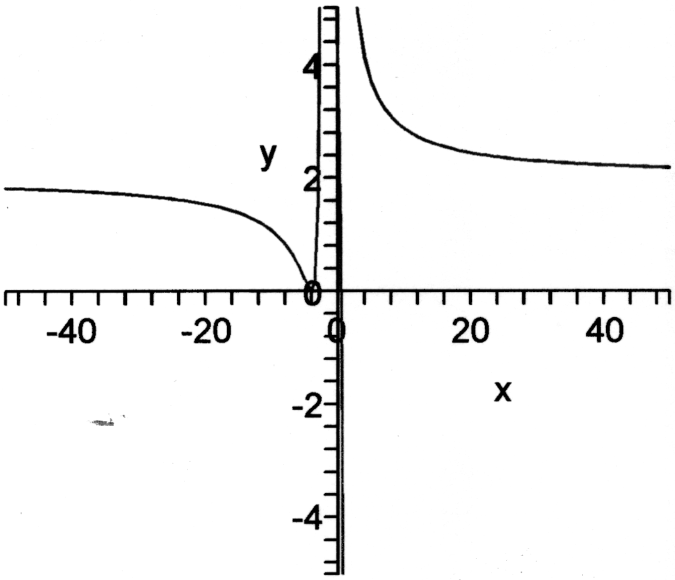
> r:=((2*x-1)*(x+4)^2)/((x-1)*(x+2)^2);
```

$$r := \frac{(2\,x - 1)\,(x+4)^2}{(x-1)\,(x+2)^2}$$

```
> plot(r,x=-5..5,y=-5..5,discont=true);
```




```
> plot(r,x=-50..50,y=-5..5,discont=true);
```

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> f:=-2*ceil(x+1)-2;
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$$f := -4 - 2\ \mathrm{ceil}(x)$$

```
> plot(f,x=-5..5,discont=true);
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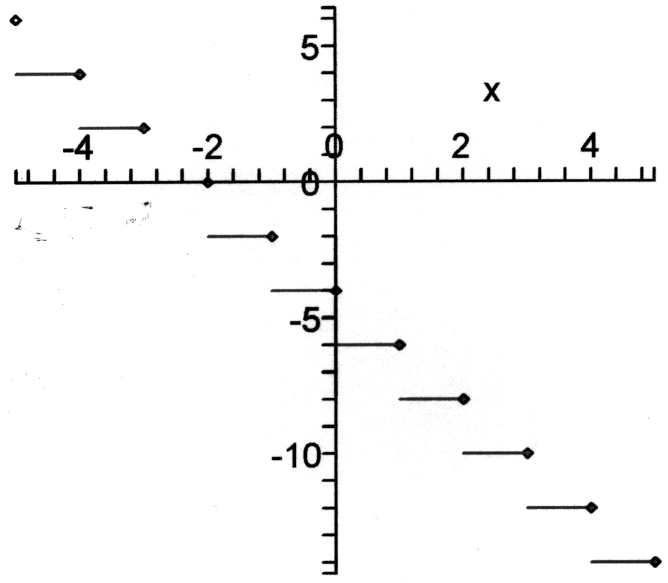
5
x
-4
-2
0
2
4
0
-5
-10

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