MATH 120A (2506) Test 1 2005 February 9

Name ANSWER KEY _____ points of 211 _____ %

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (20) Simplify:

a. $\dfrac{\frac{1}{x}-\frac{1}{x+1}}{\frac{1}{x}+\frac{1}{x+1}} = \dfrac{\left(\frac{1}{x}-\frac{1}{x+1}\right)x(x+1)}{\left(\frac{1}{x}+\frac{1}{x+1}\right)x(x+1)} = \dfrac{(x+1)-x}{(x+1)+x} = \boxed{\dfrac{1}{2x+1}}$

b. $\dfrac{2x(x+6)^4 - x^2(4)(x+6)^3}{(x+6)^8} = \dfrac{x(x+6)^3[2(x+6)-4x]}{(x+6)^8} = \dfrac{x[2x+12-4x]}{(x+6)^5}$

$= \boxed{\dfrac{x(-2x+12)}{(x+6)^5}}$

2. (15) Solve for i: $A = P\left(1+\frac{i}{100}\right)^2 \Rightarrow \frac{A}{P} = \left(1+\frac{i}{100}\right)^2 \Rightarrow \pm\sqrt{\frac{A}{P}} = 1+\frac{i}{100}$

$\Rightarrow \frac{i}{100} = -1 \pm \sqrt{\frac{A}{P}} \Rightarrow \boxed{i = -100 \pm 100\sqrt{\frac{A}{P}}}$

3. (10) Write in the form a+bi:

a. (-8+2i) - (5-3i) = $-8+2i-5+3i = \boxed{-13+5i}$

b. (-8+2i)(5-3i) = $-40+24i+10i-6i^2 = -40+34i+6 = \boxed{-34+34i}$

4. (14) Write in the form a+bi:

a. $\dfrac{-8+2i}{5-3i} = \dfrac{(-8+2i)(5+3i)}{(5-3i)(5+3i)} = \dfrac{-40-24i+10i+6i^2}{5^2+3^2} = \dfrac{-46-14i}{34} = \boxed{-\frac{46}{34}-\frac{14}{34}i}$

$= \boxed{-\frac{23}{17}-\frac{7}{17}i}$

b. $i^{315} = i^{4\cdot 78+3} = (i^4)^{78} i^3 = \boxed{-i}$

$78 = 19$
$4\overline{)315}$
$28$
$35$
$32$
$3 = R$

(59)

5. (40) Solve the equation:

a. $\left(\frac{x+1}{x}\right)^2+4\left(\frac{x+1}{x}\right)+3=0$

Let $u=\frac{x+1}{x}$

$u^2+4u+3=0$

$(u+1)(u+3)=0$

$u=-1,-3$

$\frac{x+1}{x}=-1 \Rightarrow x+1=-x \Rightarrow 2x=-1 \Rightarrow x=-1/2$

$\frac{x+1}{x}=-3 \Rightarrow x+1=-3x \Rightarrow 4x=-1 \Rightarrow x=-1/4$

$\boxed{x=-\frac{1}{2},-\frac{1}{4}}$

Check: $x=-\frac{1}{2}$: $\left(\frac{-\frac{1}{2}+1}{-\frac{1}{2}}\right)^2+4\left(\frac{-\frac{1}{2}+1}{-\frac{1}{2}}\right)+3\stackrel{?}{=}0$, $(-1)^2+4(-1)+3=0$ ✓

$x=-\frac{1}{4}$: $\left(\frac{-\frac{1}{4}+1}{-\frac{1}{4}}\right)^2+4\left(\frac{-\frac{1}{4}+1}{-\frac{1}{4}}\right)+3\stackrel{?}{=}0$, $(-3)^2+4(-3)+3=0$ ✓

b. $\frac{1}{x^3}+\frac{4}{x^2}+\frac{4}{x}=0$

$1+4x+4x^2=0$

$(1+2x)^2=0$

$\boxed{x=-1/2}$

Check: $\frac{1}{(-1/2)^3}+\frac{4}{(-1/2)^2}+\frac{4}{-1/2}\stackrel{?}{=}0$

$-8+16-8\stackrel{?}{=}0$

$0=0$ ✓

c. $2x^3+x^2-18x-9=0$

$x^2(2x+1)-9(2x+1)=0$

$(x^2-9)(2x+1)=0$

$\boxed{x=\pm 3,-1/2}$

Check: $x=3$: $2(3)^3+3^2-18(3)-9\stackrel{?}{=}0$, $54+9-54-9=0$ ✓

$x=-3$: $2(-3)^3+(-3)^2-18(-3)-9\stackrel{?}{=}0$, $-54+9+54-9=0$ ✓

$x=-\frac{1}{2}$: $2(-1/2)^3+(-1/2)^2-18(-1/2)-9\stackrel{?}{=}0$, $-2/8+1/4+9-9=0$ ✓

d. $\sqrt{x+\sqrt{x+2}}=2$

$x+\sqrt{x+2}=4$

$\sqrt{x+2}=4-x$

$x+2=(4-x)^2=16-8x+x^2$

$x^2-9x+14=0$

$(x-7)(x-2)=0$

$x=7,2$

$\boxed{x=2}$

Check: $x=7$: $\sqrt{7+\sqrt{7+2}}\stackrel{?}{=}2$, $\sqrt{7+\sqrt{9}}=\sqrt{7+3}=\sqrt{10}\neq 2$

$x=2$: $\sqrt{2+\sqrt{2+2}}\stackrel{?}{=}2$, $\sqrt{2+\sqrt{4}}=\sqrt{2+2}=\sqrt{4}=2$ ✓

**In problems #6-#8, express your answer by (1) an inequality, (2) an interval or union of intervals, and (3) a graph.**

6. (5) Solve the inequality: $4|x+2|-3<13$

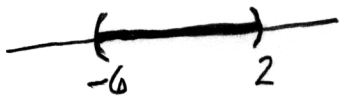
$4|x+2|<16$

$|x+2|<4$

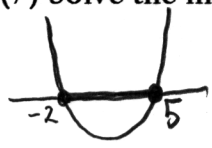
$-4<x+2<4$

$\boxed{-6<x<2}$

$\boxed{(-6,2)}$




7. (7) Solve the inequality: $x^2\le 3x+10$




$y=x^2-3x-10\le 0$

$y=(x-5)(x+2)=0$

$x=5,-2$

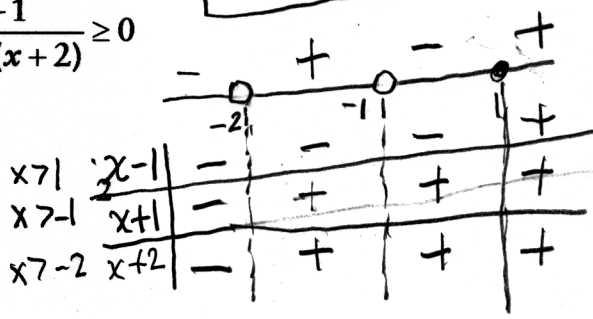
$\boxed{[-2,5]}$

$\boxed{-2\le x\le 5}$

8. (15) Solve the inequality: $y=\frac{x-1}{(x+1)(x+2)}\ge 0$

zeroes of y: $x=1$

pts. of disc. of y: $y=-1,-2$




$\boxed{-2<x<-1 \text{ or } x\ge 1}$

$\boxed{(-2,-1)\cup[1,\infty)}$

**In the remaining applied problems:**

**a. List the quantities involved; indicate the values of those that are known, and indicate those which are sought. Alternatively, prepare a table of quantities.**

**b. Make a sketch and label it (if necessary).**

**c. Write in pseudocode any relationships among the quantities.**

**d. Assign a letter to one of the quantities and represent other quantities in terms of this letter (if needed).**

**e. Write an equation needed to solve the problem.**

**f. Solve the equation.**

**g. Check your answer by reference to the original problem; explain in complete sentences why your answer is correct.**

9. (20) Sigismund earns \$7.50 per hour at his job, but if he works more than 35 hours in a week he is paid 1½ times his regular salary for the overtime hours worked. One week his gross pay was \$352.50. How many overtime hours did he work that week?

Quantities:
Basic wage rate (\$7.50/hr)
overtime wage rate
Gross pay (\$352.50)
Overtime hours (?)
Basic hours (35)

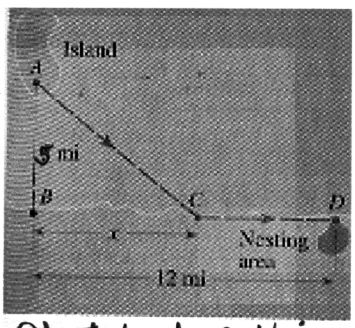
Analysis:
Overtime rate = $(1\frac{1}{2})$ basic rate
$= (1\frac{1}{2})(7.50) = \$11.25/\text{hr}$.
gross pay = Basic pay + overtime pay

Basic pay = Basic rate × Basic hours $= (7.50)(35)$
Overtime pay = OT rate × OT hours $= (11.25)(\text{OT hours})$

Equation:
Let $h$ = number of overtime hours.
$\therefore \boxed{352.50 = (7.50)(35) + 11.25h}$
$\Rightarrow \boxed{h = 8}$

Check: Basic pay $= (7.50)(35) = \$262.50$
OT pay $= (11.25)(8) = \$90.00$ +
Gross pay $= \$352.50$

10. (25) A bird is released from point A on an island, 5 miles from B, the nearest point on a straight shoreline. The bird flies to a point C on the shoreline and then flies along the shoreline to its nesting area D, as shown in the figure. The bird has 170 kcal of energy reserves; it uses 10 kcal/mi flying over land and 14 kcal/mi flying over water. Where should the point C be located so that the bird uses exactly 170 kcal of energy during its flight?




Cal rate land = 10 kcal/mi
Cal rate sea = 14 kcal/mi
Total cal = 170

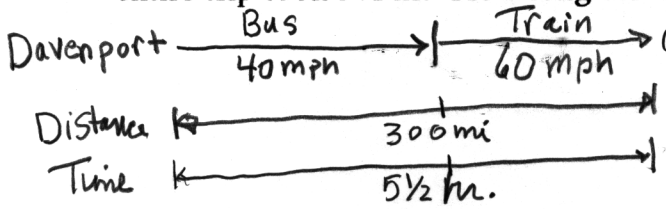
Analysis:
Total cal = cal over sea + cal over land
Cal over sea = (Cal rate)(distance) $= 14\sqrt{x^2+25}$
Cal over land = (Cal rate)(distance) $= 10(12-x)$

Equation:
$\boxed{170 = 14\sqrt{x^2+25} + 10(12-x)} \Rightarrow 25 + 5x = 7\sqrt{x^2+25}$
$\Rightarrow 25(25 + 10x + x^2) = 49(x^2+25) \Rightarrow 24x^2 - 250x + 600 = 0$
$\Rightarrow x = \frac{20}{3}, \boxed{\frac{15}{4}}$

Check: $x = 20/3$
$\overline{AC} = \sqrt{5^2 + (20/3)^2} = \frac{3}{3} \approx 5.27$
$\overline{CD} = 12 - 20/3 = 16/3$
$14\overline{AC} + 10\overline{CD} = 127.11$

$x = 15/4$
$\overline{AC} = \sqrt{5^2 + (15/4)^2} = 6\frac{1}{4}$
$\overline{CD} = 12 - 15/4 = 33/4$
$14\overline{AC} + 10\overline{CD} = 170$

11. (20) Gwendolyn took a trip from Davenport to Omaha, a distance of 300 mi. She traveled part of the way by bus, which arrived at the train station just in time for Gwen to complete her journey by train. The bus averaged 40 mi/hr and the train 60 mi/hr. The entire trip took 5½ hr. How long did Gwen spend on the train?




| | Speed | Time | Distance |
|---|---|---|---|
| Bus | 40 mph | $5.5-t$ | $40(5.5-t)$ |
| Train | 60 mph | $t$ | $60t$ |
| Total | | 5½ | 300 |

Distance = speed × time

Distance by bus + Distance by train = Total distance

Equation:

Let $t$ = time on train

$$40(5.5-t)+60t=300$$
$$\Rightarrow 220-40t+60t=300$$
$$\Rightarrow 20t=300-220$$
$$=80$$
$$\Rightarrow \boxed{t=4\text{ hr}}$$

Check: 4 hr on train $\Rightarrow 5\frac{1}{2}-4=1\frac{1}{2}$ hr on bus.

$\therefore$ Dist by bus $=(1\frac{1}{2})(40)=60$ mi

Dist by train $=4(60)=240$ mi

$\Rightarrow$ total dist $=60+240=300$ mi

12. (20) What quantity of a 60% acid solution must be mixed with a 30% solution to produce 300 mL of a 50% solution?

| | Concentration | Volume | Amt. of pure acid |
|---|---|---|---|
| Solution #1 | 60% | $s$ | $.6s$ |
| Solution #2 | 30% | $300-s$ | $.3(300-s)$ |
| Mixture | 50% | 300 | $.5(300)$ |

Amt. of pure acid = concentration × volume

Amt of acid in sol #1 + amt of acid in Sol #2 = amt of acid in mixture

Equation:

$$.6s+.3(300-s)=.5(300)$$
$$6s+900-3s=1500$$
$$3s=600$$
$$s=200$$

200 mL of 60% acid
100 mL of 30% acid

Check:

Amt. of pure acid in 60% sol $=.6(200)$
$=120$ mL

Amt of pure acid in 30% sol $=.3(100)$
$=30$ mL

$\therefore$ amt of pure acid in mixture
$=120+30=150$, which is 50%
of 300 mL!