MATH 120A (2074) **Final Test** 2003 Apr 29

Name ANSWER KEY _____ **points of 284** _____ **%**

Average with Final Test @30% **@60%** **Grade for Course**

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (10) Simplify the compound fraction:

$$\frac{\frac{1}{(a+h)^2}-\frac{1}{a^2}}{h} = \frac{(a+h)^2a^2\left[\frac{1}{(a+h)^2}-\frac{1}{a^2}\right]}{(a+h)^2a^2h} = \frac{a^2-(a+h)^2}{(a+h)^2a^2h} = \frac{a^2-(a^2+2ah+h^2)}{(a+h)^2a^2h}$$

$$= \frac{-2ah-h^2}{(a+h)^2a^2h} = \frac{-h(2a+h)}{(a+h)^2a^2h} = \boxed{-\frac{2a+h}{(a+h)^2a^2}}$$

2. (10) Solve for a: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

$$a^2b^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=a^2b^2\cdot 1 \Rightarrow b^2x^2+a^2y^2=a^2b^2 \Rightarrow b^2x^2=a^2b^2-a^2y^2$$

$$\Rightarrow b^2x^2=a^2(b^2-y^2) \Rightarrow a^2=\frac{b^2x^2}{b^2-y^2} \Rightarrow \boxed{a=\pm\sqrt{\frac{b^2x^2}{b^2-y^2}}}$$

3. (10) Write in the form a+bi:

a. $\frac{5+i}{2-3i} = \frac{(5+i)(2+3i)}{(2-3i)(2+3i)} = \frac{10+17i+3i^2}{4+9} = \boxed{\frac{7}{13}+\frac{17}{13}i}$

b. $i^{139} = i^{4\cdot 34+3} = (i^4)^{34}i^3 = i^3 = \boxed{-i}$

$$4\overline{)139}\quad 34,\ 12,\ 19,\ 16,\ 3$$

4. (40) Solve the equations:

a. $\sqrt{x}-3\sqrt[4]{x}-4=0$ Let $u=\sqrt[4]{x} \Rightarrow u^2=\sqrt{x}$

$u^2-3u-4=0$ | $u=4,-1$ | $\boxed{x=256}$

$(u-4)(u+1)=0$ | $\sqrt[4]{x}=4,-1$ |

Check: $x=256$: $\sqrt{256}-3\sqrt[4]{256}-4 \stackrel{?}{=} 0$; $16-3\cdot 4-4=0$

$x=1$: $\sqrt{1}-3\sqrt[4]{1}-4 \stackrel{?}{=} 0$; $1-3-4\neq 0$

b. $\sqrt{x+\sqrt{x+2}}=2$

$x+\sqrt{x+2}=4$

$\sqrt{x+2}=4-x$

$x+2=16-8x+x^2$

$x^2-9x+14=0$

$(x-7)(x-2)=0$

$\boxed{x=2}$

Check: $x=7$: $\sqrt{7+\sqrt{9}} \stackrel{?}{=} 2$; $\sqrt{7+3}\neq 2$

$x=2$: $\sqrt{2+\sqrt{4}} \stackrel{?}{=} 2$; $\sqrt{2+2}=2$

5. (20) Solve the inequality: $\frac{(x^2+1)(x-3)}{x^2-9}\geq 0$.

Real roots of numerator: 3

Real roots of denominator: ± 3

	-3		3	
x^2+1	+	+		+
$x-3$	−	−		+
$x-3$	−	−		+
$x+3$	−	+		+
90°	−	+		+

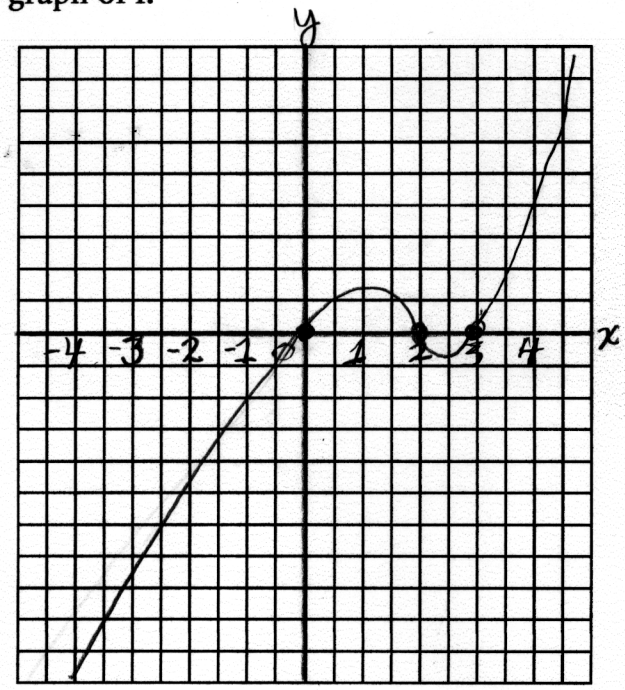
$\boxed{x>-3,\neq 3}$

$\boxed{(-3,3)\cup(3,\infty)}$

6. (15) For the polynomial function $f(x)=x^3-5x^2+6x$

a. (3) Approximate f(x) by a power function: $f(x) \approx x^3$ for $|x|$ large

b. (12) Plot the graph of f.

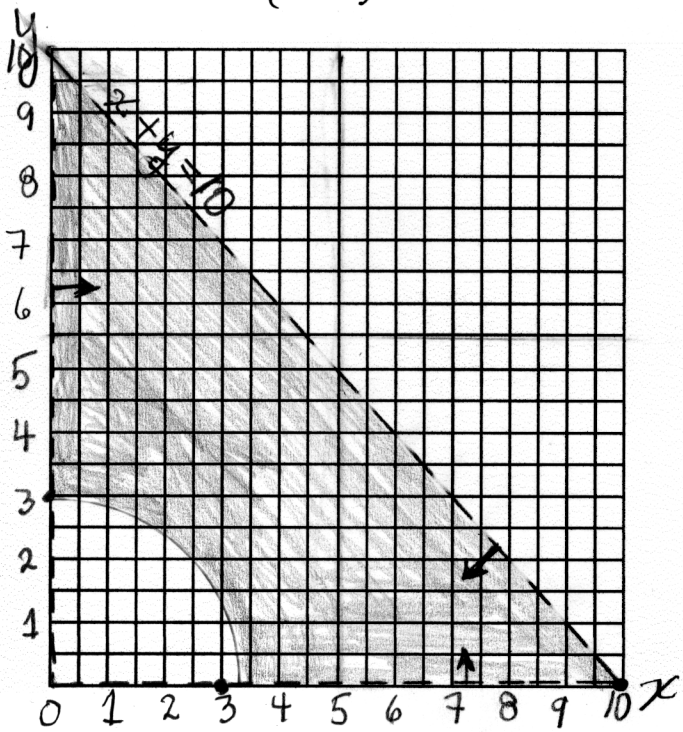


$f(x) = x(x^2-5x+6)$

$= x(x-3)(x-2)$

		0	2	3
x	−	+	+	+
$x-3$	−	−	−	+
$x-2$	−	−	+	+
f	−	+	−	+

7. (15) Solve the system of inequalities: $\begin{cases} x>0 \\ y>0 \\ x+y<10 \\ x^2+y^2>9 \end{cases}$ $x+y=10 \Leftrightarrow y=10-x$



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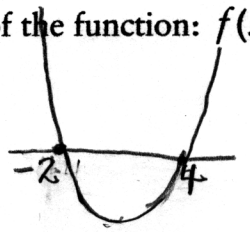
8. (12) Find the domain of the function: $f(x) = \dfrac{x^2}{\sqrt{x^2-2x-8}}$.

$x^2-2x-8>0$

$(x-4)(x+2)>0$

parabola opens up



$x<-2$ or $x>4$

$(-\infty,-2)\cup(4,\infty)$

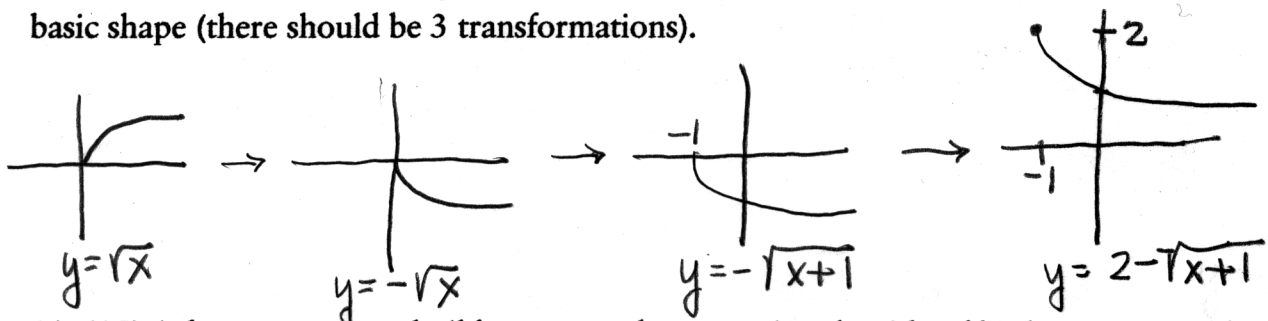
9. (15) The heat experienced by a hiker at a campfire is proportional to the amount of wood on the fire, and inversely proportional to the cube of his distance from the fire. If he is 20 ft from the fire, and someone doubles the amount of wood burning, how far from the fire would he have to be so that he feels the same heat as before?

$H = k\frac{W}{D^3}$; $D=20 \Rightarrow H = \frac{kW}{20^3}$. $W'=2W \Rightarrow H' = k\frac{W'}{(D')^3} = k\frac{2W}{(D')^3}$

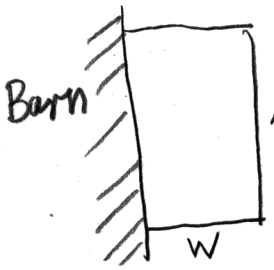
If $H=H'$, then $k\frac{2W}{(D')^3} = \frac{kW}{20^3} \Rightarrow \frac{2}{(D')^3} = \frac{1}{20^3} \Rightarrow 2\cdot 20^3 = (D')^3 \Rightarrow D' = \boxed{20\sqrt[3]{2}}$

$= 25.198$

10. (12) Sketch the graph of $y = 2-\sqrt{x+1}$ by a sequence of transformations from a basic shape (there should be 3 transformations).



11. (15) A farmer wants to build a rectangular pen using the side of his barn as one side of the pen. He has 32 ft of fencing. What are the dimensions of the pen to enclose the maximum area, and what is this maximum area?



Total length of fencing $= 32 = 2w+l \Rightarrow l = 32-2w$

Area $= wl$

To maximize $A = w(32-2w)$

$= -2w^2+32w$

Parabola opens downward, $\therefore \exists$ maximum.

maximum at $w = -\frac{b}{2a} = -\frac{32}{2(-2)} = 8$

Dimensions of pen: $\boxed{8 \times 16}$

Area of pen $= 8\cdot 16 = \boxed{128}$

12. (15) Use either the Gauss or the Gauss-Jordan method to solve the following system of linear equations:

$$\begin{cases} 2x-y+4z=-3 \\ x-2y-10z=-6 \\ 3x+4z=7 \end{cases}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 4 & -3 \\ 1 & -2 & -10 & -6 \\ 3 & 0 & 4 & 7 \end{array}\right) \to \left(\begin{array}{ccc|c} \textcircled{1} & -2 & -10 & -6 \\ 2 & -1 & 4 & -3 \\ 3 & 0 & 4 & 7 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -2 & -10 & -6 \\ 0 & 3 & 24 & 9 \\ 0 & 6 & 34 & 25 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & -2 & -10 & -6 \\ 0 & 1 & 8 & 3 \\ 0 & 6 & 34 & 25 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 1 & 8 & 3 \\ 0 & 0 & -14 & 7 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 1 & 8 & 3 \\ 0 & 0 & 1 & -1/2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1/2 \end{array}\right)$$

$\boxed{\begin{array}{l} x=3 \\ y=7 \\ z=-1/2 \end{array}}$

13. (20) A jeweler has three small solid spheres made of gold, of radius 2 mm, 3 mm, and 4 mm. He decides to melt these down and make just one sphere out of them. What will the radius of this larger sphere be? ($V = \frac{4}{3}\pi r^3$)

Total volume of gold = vol in sphere #1 + vol in sphere #2 + vol in sphere #3
$= \frac{4}{3}\pi\cdot 2^3 + \frac{4}{3}\pi\cdot 3^3 + \frac{4}{3}\pi\cdot 4^3 = \frac{4}{3}\pi(2^3+3^3+4^3) = \frac{4}{3}\pi(99)$

Total volume of gold = volume in resulting sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(99) \Rightarrow r^3 = 99 \Rightarrow \boxed{r = \sqrt[3]{99} = 4.626}$

14. (25) A group of friends decides to buy a vacation home for \$120,000, sharing the cost equally. If they could find one more person to join them, each person's contribution would drop by \$6000. How many people are in the group?

Quantities: cost of home (\$120,000)
no. of buyers (?)
individual contribution
Analysis: Individual contribution = $\frac{\text{cost of home}}{\text{no. of buyers}}$

When no. of buyers increases by 1, individual contribution decreases by \$6000

Let n = no. of buyers

$\boxed{\frac{120{,}000}{n+1} = \frac{120{,}000}{n} - 6000}$

$\frac{20}{n+1} = \frac{20}{n} - 1$

$20n = 20(n+1) - n(n+1)$

$20n = 20n + 20 - n^2 - n$

$n^2 + n - 20 = 0$

$(n+5)(n-4) = 0$

$\boxed{n = 4}$

check: 4 buyers $\Rightarrow$ ind. contrib = \$30,000
5 buyer $\Rightarrow$ ind contrib = \$24000

15. (20) Henry and Irene working together can wash all the windows of their house in 1 h 48 min. Working alone, it takes Henry 1½ h more than Irene to do the job. How long does it take each person working alone to wash all the windows.

Quantities: Time for Henry to do job
Time for Irene to do job
Time for both to do job
Analysis: Time for Henry = Time for Irene + 1½

Time for both together = 1.8h
Let I = time for Irene
Then I + 3/2 = time for Henry
Rate for Irene + rate for Henry = Joint Rate

$\boxed{\frac{1}{I} + \frac{1}{I+3/2} = \frac{1}{1.8}}$

$1.8(I+3/2) + 1.8I = I(I+3/2)$

$3.6I + 2.7 = I^2 + 1.5I$

$I^2 - 2.1I - 2.7 = 0$

$\boxed{I = 3,\ H = 4.5}$

check: $\frac{1}{3} + \frac{1}{4.5} = \frac{1}{1.8}$

16. (20) The fuel consumption for William's car is 30 mi/gal on the highway and 25 mi/gal in the city. On a vacation trip of 400 mi he used 14 gal of gasoline. How many highway miles did he drive on this trip?

Quantities: Rate of fuel cons. on highway (30 mi/gal)
Rate of fuel cons. in city (25 mi/gal)
Miles on highway
Miles in city
Total miles (400)
gas on highway
gas in city
Total gas (14)

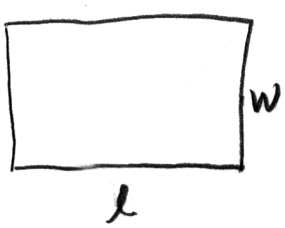
Analysis: gas on highway + gas in city = total gas
gas in city (hours) = miles / rate
Let c = gas in city (gals)
h = gas in highway (gals)

$\begin{cases} c + h = 14 \\ 25c + 30h = 400 \end{cases}$

$25c + 30h = 400$
$25c + 25h = 350$
$5h = 50$
$h = 10,\ c = 4$

Mi on hgwy = 300, mi in city = 100

17. (10) A garden has a perimeter of 39 ft. The length is twice the width. Find the dimensions of the garden.



Perimeter = 2(length + width)
Length = 2(width)
Let w = width of garden (in ft.)
Then 2w = length of garden (in ft.)

$2(2w + w) = 39$
$6w = 39$
$w = 13/2$

$\boxed{\text{width} = 13/2 \text{ ft}}$
$\boxed{\text{length} = 13 \text{ ft}}$

check: $2\cdot\frac{13}{2} + 2\cdot 13 \stackrel{?}{=} 39$
$13 + 26 = 39$