MATH 120A (2074) **Test 3** 2003 Apr 24

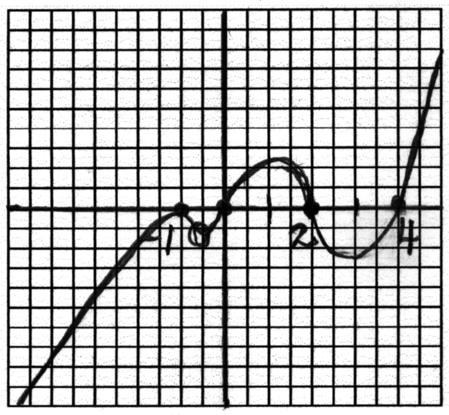
Name ANSWER KEY ______points of 125 ______%

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (15) (a) (3) Approximate $f(x)=x^3(x+1)^2(x-2)(x-4)$ by a power function: $f(x)\approx x^3 \cdot x^2 \cdot x \cdot x = \boxed{x^7}$

(b) (12) Sketch the graph of f(x).

zeroes of f: 0, -1, 2, 4



	-1	0	2	4	
x^3	−	−	+	+	+
$(x+1)^2$	+	+	+	+	+
$x-2$	−	−	−	+	+
$x-4$	−	−	−	−	+
f	−	−	+	−	+

2. (5) Find the quotient and remainder if $f(x)=3x^3-5x^2-4x-8$ is divided by $p(x)=2x^2+x$.

$$\begin{array}{r} \boxed{\tfrac{3}{2}x-\tfrac{13}{4}} \\ 2x^2+x\,\big)\,3x^3-5x^2-4x-8 \\ 3x^3+\tfrac{3}{2}x^2 \\ \hline -\tfrac{13}{2}x^2-4x-8 \\ -\tfrac{13}{2}x^2-\tfrac{13}{4}x \end{array}$$

$R = \boxed{-\frac{3}{4}x-8}$

3. (10) Solve by the method of substitution:

$$\left\{\begin{array}{l} xy=2 \\ 3x-y+5=0 \Rightarrow y=3x+5 \end{array}\right\} \Rightarrow x(3x+5)=2 \Rightarrow 3x^2+5x-2=0$$

$3x^2+6x-x-2=0$

$3x(x+2)-(x+2)=0$

$(3x-1)(x+2)=0$

$x=\frac{1}{3},-2$

$\boxed{(1/3,6)\ (-2,-1)}$

4. (15) Solve by employing either the Gauss-Jordan method or the Gauss method:

$$\begin{cases} x+2y-2z=-1 \\ 3x-y+2z=7 \\ 5x+3y-4z=2 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 3 & -1 & 2 & 7 \\ 5 & 3 & -4 & 2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & -7 & 8 & 10 \\ 0 & -7 & 6 & 7 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & -7 & 8 & 10 \\ 0 & 0 & -2 & -3 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & -7 & 0 & -2 \\ 0 & 0 & -2 & -3 \end{array}\right) \quad \begin{cases} x+2y = 2 \\ -7y = -2 \\ -2z=-3 \end{cases} \Rightarrow \boxed{\begin{array}{l} y=\frac{2}{7} \\ z=3/2 \end{array}}$$

$x+2(\frac{2}{7})=2$

$x=2-\frac{4}{7}$

$\boxed{x=\frac{10}{7}}$

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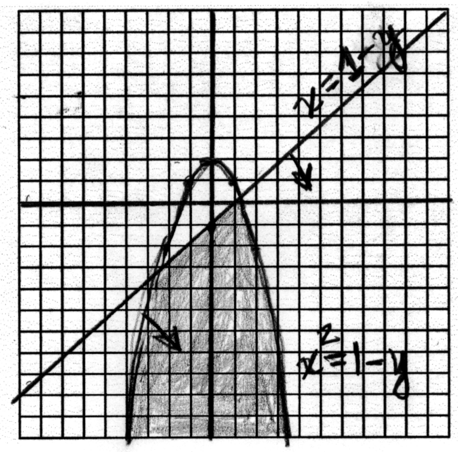
5. (10) Sketch the graph of the system of inequalities: $\begin{cases} x^2 \le 1-y \Rightarrow y \le 1-x^2 \\ x > 1+y \Rightarrow y < x-1 \end{cases}$



6. (20) *A linear programming problem:* A company manufactures two types of electric hedge-trimmers, one of which is cordless. The cord-type trimmer requires 2 hours of labor to make and the cordless model requires 4 hours. The company has at most 800 hours of labor to use in manufacturing each day, and the packaging department can package no more than 300 trimmers per day. If the profit of each cord-type model is \$320 and the profit of each cordless model is \$40, how many of each type should be manufactured per day to maximize the profit? What is the maximum daily profit?

	Labor	No.	Profit Rate	Profit
Cord-type	2	x	320	$320x$
Cordless	4	y	40	$40y$

a. (3) What is the objective function?

$\boxed{P = 320x + 40y}$

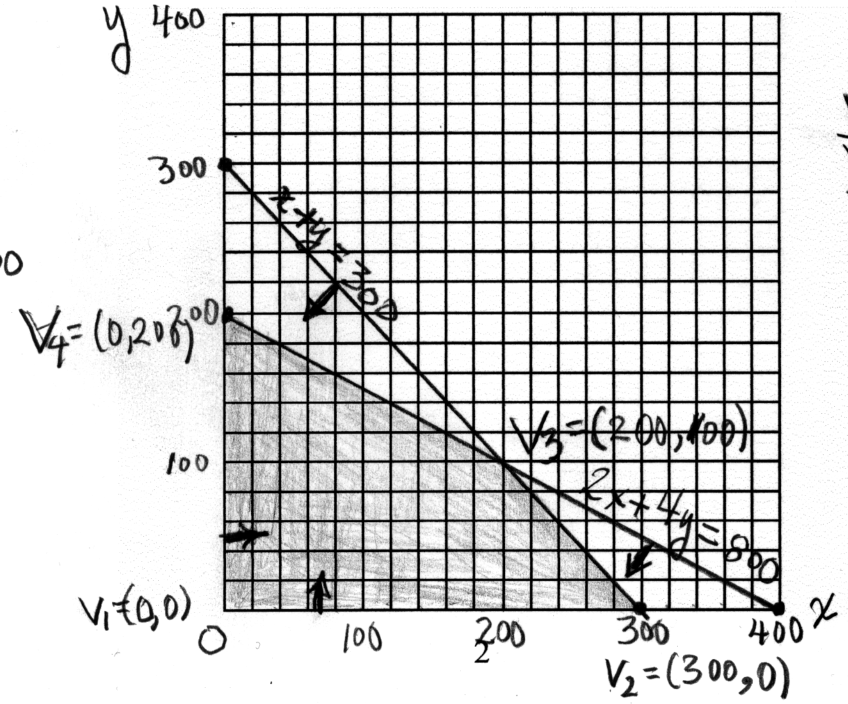
b. (5) Give the constraints.

$0 \le x$
$0 \le y$
$2x + 4y \le 800 \Rightarrow x + 2y \le 400$
$x + y \le 300$

Hours of Labor ≤ 800
$2x + 4y \le 800$
No. of trimmers ≤ 300
$x + y \le 300$

c. (12) Solve the linear programming problem.

$\begin{cases} x + 2y = 400 \\ x + y = 300 \end{cases}$
$y = 100$
$x = 300 - y = 200$



Vertex	$P = 320x + 40y$
V_1	0
V_2	96 000
V_3	68 000
V_4	8000

To maximize profit, 300 cord-type 0 cordless should be produced, for a profit of \$96 000 per day.

7. (25) a. (10) Use determinants to show that $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$ has an inverse.

$$\begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 6 & 0 & 11 \end{vmatrix} = -1 \begin{vmatrix} 1 & 2 \\ 6 & 11 \end{vmatrix} = -(11-12) = -(-1) = 1 \neq 0$$

b. (15) Find $A^{-1} = \boxed{\begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}}$

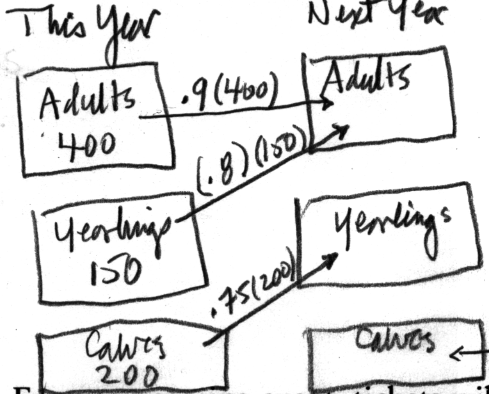
$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & -1 & -1 & -2 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array}\right)$$

8. (15) **Set up only the system of equations for solving the following problem.** ***Do not solve it.***

A rancher has 750 head of cattle consisting of 400 adults (aged 2 or more years), 150 yearlings, and 200 calves. The following information is known about this particular species. Each spring an adult female gives birth to a single calf, and 75% of these calves will survive the first year. The yearly survival percentages for yearlings and adults are 80% and 90%, respectively. The male-female ratio is one in all age classes. Estimate the population of each age class next spring.

	Initial Pop.	Survival Rate
Adults	400	90%
Yearlings	150	80%
Calves	200	75%

Let A = No. of adults next year
Y = No. of yearlings next year
C = No. of calves next year



$A = (.9)(400) + (.8)(150)$

$Y = (.75)(200)$

$C = \frac{1}{2}[(.9)(400) + (.8)(15$

9. (10) An auditorium contains 600 seats. For an upcoming event, tickets will be priced at \$8 for some seats and \$5 for others. At least 225 tickets are to be priced at \$5, and total sales of at least \$3000 are desired. Find a system of inequalities that describes all possibilities for pricing the two types of tickets.

Total number of tickets ≤ 600
Number of \$5 tickets ≥ 225
Total sales ≥ 3000

Let E = number of \$8 tickets
F = number of \$5 tickets

$$\begin{cases} E \geq 225 \\ F \geq 0 \\ E + F \leq 600 \\ 8E + 5F \geq 3000 \end{cases}$$

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