MATH 120A (2410) **Final Test** 2002 Dec 10

Name ANSWER KEY _____ **points of 316** _____ **%**

Average with Final Test @30% **@60%** **Grade for Course**

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (10) Simplify the compound fraction:

$$\frac{x^{-1}+y^{-1}}{(x+y)^{-1}} = \frac{\left(\frac{1}{x}+\frac{1}{y}\right)xy(x+y)}{\frac{1}{x+y}\cdot xy(x+y)} = \frac{(y+x)(x+y)}{xy} = \boxed{\frac{(x+y)^2}{xy}}$$

2. (10) Solve for r: $F = G\frac{mM}{r^2}$.

$$\Rightarrow r^2F = GmM \Rightarrow r^2 = \frac{GmM}{F} \Rightarrow \boxed{r = \pm\sqrt{\frac{GmM}{F}}}$$

3. (10) Write in the form a+bi:

a. $\frac{1+2i}{3+4i} = \frac{(1+2i)(3-4i)}{(3+4i)(3-4i)} = \frac{3+8i^2+6i-4i}{3^2+4^2} = \frac{11+2i}{9+16} = \boxed{\frac{11}{25}+\frac{2}{25}i}$

b. $i^{67} = i^{(4\cdot16+3)} = (i^4)^{16}\cdot i^3 = 1^{16}(-i) = \boxed{-i}$

4. (40) Solve the equations:

a. $2x = 1-\sqrt{2-x}$

$2x-1 = -\sqrt{2-x}$

$4x^2-4x+1 = 2-x$

$4x^2-3x-1=0$

$x = \frac{3\pm\sqrt{9+4\cdot4}}{2\cdot4} = \frac{3\pm\sqrt{25}}{8} = \frac{3\pm5}{8} = \cancel{1}, \boxed{-\frac{1}{4}}$

Check: $x=1$: $2\cdot1 \stackrel{?}{=} 1-\sqrt{2-1}$, $2 \stackrel{?}{=} 1-1$, $2 \neq 0$

$x=-\frac{1}{4}$: $2(-\frac{1}{4}) \stackrel{?}{=} 1-\sqrt{2+\frac{1}{4}}$, $-\frac{1}{2} \stackrel{?}{=} 1-\sqrt{\frac{9}{4}}$, $-\frac{1}{2} \stackrel{?}{=} 1-\frac{3}{2}$, $-\frac{1}{2} = -\frac{1}{2}$

b. $\left(1+\frac{1}{x}\right)^2 - 6\left(1+\frac{1}{x}\right)+8=0$

$w^2-6w+8=0$

$(w-2)(w-4)=0$

$w=2,4$

$1+\frac{1}{x} = 2,4$

$\frac{1}{x} = 1,3$

$\boxed{x=1,\ \frac{1}{3}}$

Check:

$x=1$: $(1+1)^2-6(1+1)+8 \stackrel{?}{=} 0$, $4-12+8 \stackrel{?}{=} 0$, $0=0$

$x=\frac{1}{3}$: $(1+3)^2-6(1+3)+8 \stackrel{?}{=} 0$, $16-24+8 \stackrel{?}{=} 0$, $0=0$

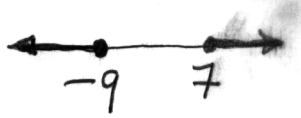
5. (15) Solve the inequality: $\left|\frac{x+1}{2}\right| \geq 4$.

$\frac{|x+1|}{2} \geq 4$

$|x+1| \geq 8$

$x+1 \leq -8$ or $x+1 \geq 8$

$\boxed{x \leq -9 \text{ or } x \geq 7}$ or $\boxed{(-\infty,-9]\cup[7,\infty)}$

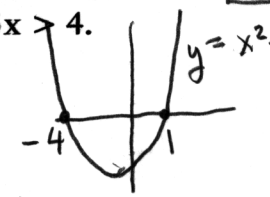


6. (10) Solve the inequality: $x^2+3x > 4$.

$y = x^2+3x-4 > 0$

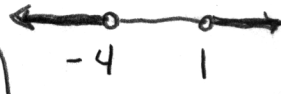
Parabola opens up.

x-intercepts: $(x+4)(x-1)=0 \Rightarrow x=1,-4$.



$\boxed{x<-4 \text{ or } x>1}$

$\boxed{(-\infty,-4)\cup(1,\infty)}$

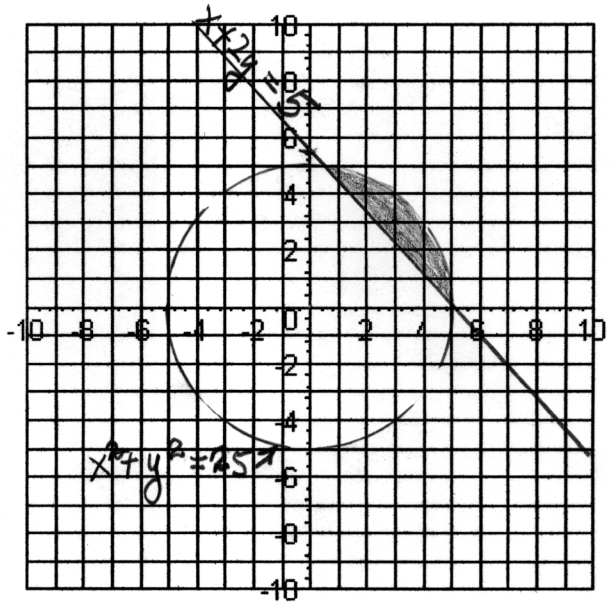


7. (15) Solve the system of inequalities. Graph your solution set.

$$\begin{cases} x^2 + y^2 < 25 \\ x + 2y \geq 5 \end{cases}$$

$x + 2y \geq 5 \Rightarrow 2y \geq -x + 5$

$\Rightarrow y \geq -\frac{1}{2}x + \frac{5}{2}$



8. (20) Solve the inequality: $\dfrac{(x-1)^2}{(x+1)(x+2)} > 0$.

$(x-1)^2 \neq 0 \Rightarrow x \neq 1$

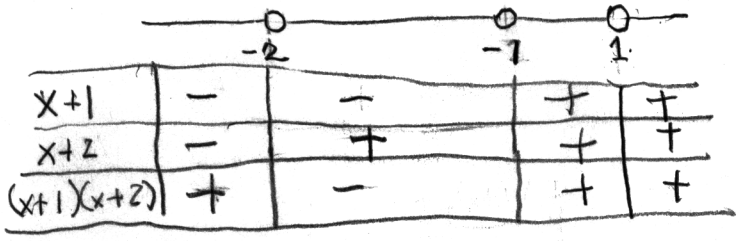
$(x+1)(x+2) = 0 \Rightarrow x = -1, -2$

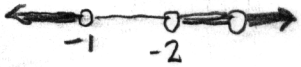
$x+1 > 0 \Rightarrow x > -1$

$x+2 > 0 \Rightarrow x > -2$

$x < -2$ or $x > -1$ but $x \neq 1$

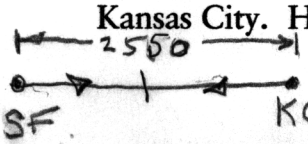
$(-\infty, -2) \cup (-1, 1) \cup (1, \infty)$





Solve the applied problems #9-#12. (1) Show your preliminary analysis. (2) State your equation(s), labeling your unknowns. (3) Solve the equation. (4) Check your result by reference to the stated problem.

9. (20) A jet took off from Kansas City for San Francisco, a distance 2550 km, at a speed of 800 km/h. At the same time a private jet, traveling at 900 km/h, left San Francisco for Kansas City. How long after takeoff will the jets pass each other?



DISTANCE = RATE × TIME

	RATE	TIME	DIST
KC → SF	800	t	$800t$
SF → KC	900	t	$900t$
TOTAL	×	×	2550

DIST. OF KC→SF + DIST OF SF→KC = 2550

$800t + 900t = 2550$

$1700t = 2550$

$t = \frac{2550}{1700} = \boxed{1.5 \text{ hr.}}$

Check: KC → SF dist $= 1.5 \times 800 = 1200$

SF → KC dist $= 1.5 \times 900 = 1350$

2550 = TOTAL

10. (25) Betty and Karen have been hired to paint the houses in a new development. Working together the women can paint a house in two-thirds the time that it takes Karen working alone. Betty takes 6 h to paint a house alone. How long does it take Karen to paint a house working alone?

	Time	Rate
Betty	6	$1/6$
Karen	k	$1/k$
Together	$\frac{2}{3}k$	$\frac{3}{2k}$

Rate of Betty + Rate of Karen = Rate together

$\frac{1}{6} + \frac{1}{k} = \frac{3}{2k}$

$k + 6 = 3 \cdot 3$

$k = 9 - 6 = \boxed{3}$

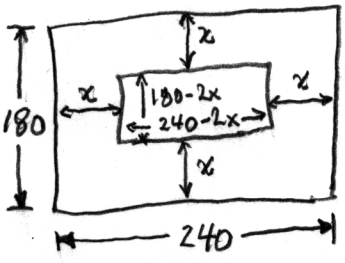
Check: Rate of $K = \frac{1}{3}$

Rate together $= \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{1}{2}$,

which implies time together $= 2$,

which is $\frac{2}{3} \cdot 3$.

11. (20) A factory is to be built on a lot measuring 180 ft by 240 ft. The local building code specifies that a lawn of uniform width and equal in area to the factory must surround the factory. What must the width of this lawn be, and what are the dimensions of the factory?



From the diagram: Area of Factory $= (180-2x)(240-2x)$

Area of lawn = area of lot − area of factory $= 180 \cdot 240 - (180-2x)(240-2x)$

Area of lawn = area of factory. $\therefore (180-2x)(240-2x) = 180\cdot 240 - (180-2x)(240-2x)$

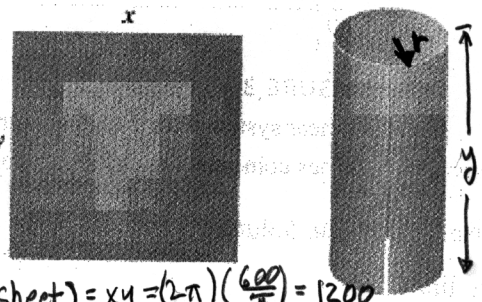
$\Rightarrow 2(180-2x)(240-2x) = 180\cdot 240 \Rightarrow 43200 - 840x + 4x^2 = 21600$

$\Rightarrow x^2 - 210x + 5400 = 0 \Rightarrow x = \frac{210 \pm \sqrt{210^2 - 4\cdot 1\cdot 5400}}{2} = \frac{210 \pm 150}{2} = 180, 30$

width of lawn = **30** dimensions of factory = 180×120 Check: $A(\text{factory}) = 21600$

$A(\text{lawn}) = 180\cdot 240 - 21600 = 21600$

12. (25) A rectangular piece of sheet metal with an area of 1200 in^2 is to be bent into a cylindrical length of stovepipe having a volume of 600 in^3. What are the dimensions of the sheet metal?



Circumference of pipe $= 2\pi r$

circumference of pipe $= x$

$\Rightarrow x = 2\pi r \Rightarrow r = \frac{x}{2\pi}$

Volume of pipe $= \pi r^2 h = \pi\left(\frac{x}{2\pi}\right)^2 y$

Area of sheet $= xy$

$\therefore \begin{cases} \pi\left(\frac{x}{2\pi}\right)^2 y = 600 \\ xy = 1200 \end{cases} \Rightarrow \begin{cases} x^2 y = 2400\pi \\ xy = 1200 \end{cases} \Rightarrow x(1200) = 2400\pi$

$x = 2\pi$

$y = \frac{1200}{x} = \frac{1200}{2\pi} = \frac{600}{\pi} = y$

Check: $A(\text{sheet}) = xy = (2\pi)\left(\frac{600}{\pi}\right) = 1200$

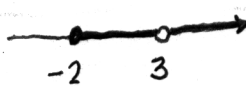
$V(\text{pipe}) = \pi r^2 h = \pi 1^2 \cdot \frac{600}{\pi} = 600$

13. (6) What is the domain of $f(x) = \dfrac{\sqrt{2+x}}{x-3}$?

(1) $2+x \geq 0 \Rightarrow x \geq -2$

(2) $x-3 \neq 0 \Rightarrow x \neq 3$

$\therefore$ Domain $= \{x \mid x \geq -2, x \neq 3\} = [-2, 3) \cup (3, \infty)$



14. (10) The period of a pendulum (the time elapsed during one complete swing of the pendulum) varies directly with the square root of the length of the pendulum.

(a) (4) Express this relationship by writing an equation.

$P = k\sqrt{l}$ where P = period of pendulum, l = length of pendulum.

(b) (6) In order to double the period, how would we have to change the length of the pendulum?

$P' = 2P = 2k\sqrt{l}$

$P' = k\sqrt{l'}$

$\therefore 2k\sqrt{l} = k\sqrt{l'} \Rightarrow 2\sqrt{l} = \sqrt{l'} \Rightarrow l' = 4l$

15. (20) The graph of y=f(x) is given. Match each equation with its graph.

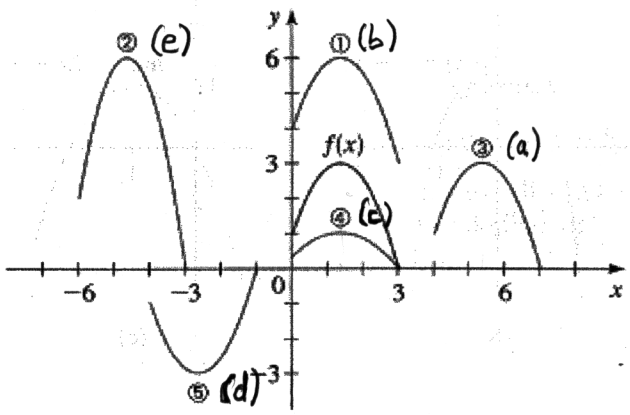
(a) y=f(x-4) (3)

(b) y=f(x)+3 (1)

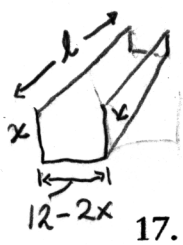
(c) y=⅓f(x) (4)

(d) y=-f(x+4) (5)

(e) y=2f(x+6) (2)



16. (20) A long rectangular sheet of metal, 12 in wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?



From the figure, $V = l(12-2x)x = -2lx^2 + 12lx$, quadratic function whose parabola opens down and which ∴ has a maximum at $x = -\frac{b}{2a}$

$= -\frac{12l}{2(-2l)} = -\frac{12l}{-4l} = \boxed{3}$

17. (5) Is $f(x) = x^2+1$ even, odd, or neither?

$f(-x) = (-x)^2+1 = x^2+1 = f(x)$ ∴ **f is even**

18. (15) Use either the Gauss-Jordan method or the Gauss method to solve:

$$\begin{cases} x+2y-3z-4w=10 \\ x+3y-3z-4w=15 \\ 2x+2y-6z-8w=10 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 10 \\ 1 & 3 & -3 & -4 & 15 \\ 2 & 2 & -6 & -8 & 10 \end{array}\right) \to \left(\begin{array}{cccc|c} \textcircled{1} & 2 & -3 & -4 & 10 \\ 1 & 3 & -3 & -4 & 15 \\ 1 & 1 & -3 & -4 & 5 \end{array}\right)$$

$\frac{1}{2}\textcircled{3}$

$$\to \left(\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 10 \\ 0 & \textcircled{1} & 0 & 0 & 5 \\ 0 & -1 & 0 & 0 & -5 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & -3 & -4 & 0 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x - 3z - 4w = 0 \\ y = 5 \end{cases} \Rightarrow \boxed{\begin{cases} x = 3z+4w \\ y = 5 \\ z = \text{arbitrary} \\ w = \text{arbitrary} \end{cases}}$$

$\textcircled{2}-\textcircled{1}\to\textcircled{2}$ $\quad$ $\textcircled{1}-2\textcircled{2}\to\textcircled{1}$

$\textcircled{3}-\textcircled{1}\to\textcircled{3}$ $\quad$ $\textcircled{2}+\textcircled{3}\to\textcircled{3}$

19. (20) Set up only the system equations for solving the following problem. ***Do not solve it.***

A nutritionist wishes to feed one of the subjects in his experiment a daily diet that consists of a combination of three commercial diet foods: MiniCal, SloStarve, and SlimQuick. For the experiment it's important that the subject consume exactly 500 mg of potassium, 75 gr of protein, and 1150 units of vitamin D every day. The amounts of these nutrients in one ounce of each food are given in the table. How many ounces of each food should the subject eat every day to satisfy the nutrient requirements exactly?

	MiniCal	SloStarve	Slimquick
Potassium (mg)	50	75	10
Protein (g)	5	10	3
Vitamin D (units)	90	100	50

Pot. from Minical + Pot. from SloStarve + Pot. from Slimquick = Total Pot.

Similarly for protein and vitamin D.

Let M = no. of ounces of Minical/day

S = " " " " SloStarve "

Q = " " " " Slimquick "

$$\begin{cases} 50M + 75S + 10Q = 500 \\ 5M + 10S + 3Q = 75 \\ 90M + 100S + 50Q = 1150 \end{cases}$$