MATH 120A (2410) **Test 3** 2002 Dec 5

Name ANSWER KEY ______points of 150 ______%

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (20) Solve the system by the method of substitution: $\begin{cases} x^2+z^2=5 \\ 2x+y=1 \\ y+z=1 \end{cases}$ $\Rightarrow 2x-z=0 \Rightarrow z=2x$

②−③

$\therefore x^2+(2x)^2=5 \Rightarrow 5x^2=5 \Rightarrow x^2=1$

$\therefore x=\pm1$

$y=1-2x=1-2(\pm1)=1\mp2$

$z=1-y=1-(1\mp2)=1-1\pm2=\pm2$

$\boxed{\begin{matrix}(1,-1,2)\\(-1,3,-2)\end{matrix}}$

Check: $(1,-1,2)$: $\begin{cases}1^2+2^2=1+4=5\\2\cdot1+(-1)=1\\-1+2=1\end{cases}$

$(-1,3,-2)$: $\begin{cases}(-1)^2+(-2)^2=1+4=5\\2(-1)+3=1\\3+(-2)=1\end{cases}$

2. (15) Use either the Gauss-Jordan method or the Gauss method to solve: $\begin{cases} x+y-5z=3 \\ x \quad\; -2z=1 \\ 2x-y-z=0 \end{cases}$

$$\left(\begin{array}{ccc|c}1&1&-5&3\\1&0&-2&1\\2&-1&-1&0\end{array}\right) \xrightarrow[-2①+③]{-①+②} \left(\begin{array}{ccc|c}1&1&-5&3\\0&-1&3&-2\\0&-3&9&-6\end{array}\right) \xrightarrow[-\frac{1}{3}③]{-1②} \left(\begin{array}{ccc|c}1&1&-5&3\\0&1&-3&2\\0&1&-3&2\end{array}\right)$$

$$\xrightarrow[-②+③]{①-②} \left(\begin{array}{ccc|c}1&0&-2&1\\0&1&-3&2\\0&0&0&0\end{array}\right) \quad \begin{cases}x-2z=1\\y-3z=2\end{cases} \Rightarrow \boxed{\begin{cases}x=1+2z\\y=2+3z\\z \text{ arbitrary}\end{cases}} \text{ or } \boxed{\begin{cases}x=1+2t\\y=2+3t\\z=t\end{cases} \quad t \text{ arb}}$$

3. (10) Multiply the two matrices:

$$\begin{pmatrix}1&2&-3\\4&-5&6\end{pmatrix}\begin{pmatrix}1&-1&0&2\\-2&3&1&0\\0&4&0&-3\end{pmatrix}=$$

2×③ ③×4

$\therefore$ 2×4

$$=\begin{pmatrix}1\cdot1+2(-2)-3\cdot0 & 1(-1)+2\cdot3-3\cdot4 & 1\cdot0+2\cdot1-3\cdot0 & 1\cdot2+2\cdot0-3(-3)\\4\cdot1-5(-2)+6\cdot0 & 4(-1)-5\cdot3+6\cdot4 & 4\cdot0-5\cdot1+6\cdot0 & 4\cdot2-5\cdot0+6(-3)\end{pmatrix}$$

$$=\begin{pmatrix}1-4 & -1+6-12 & 2 & 2+9\\4+10 & -4-15+24 & -5 & 8-18\end{pmatrix}=\boxed{\begin{pmatrix}-3&-7&2&11\\14&5&-5&-10\end{pmatrix}}$$

4. (5) Evaluate $\det\begin{pmatrix}1.2&3.1&-7.9&13.1\\0.7&-5.7&-1.4&0.3\\1.2&3.1&-7.9&13.1\\9.9&1.2&-0.1&6.3\end{pmatrix}$ without much effort. *Justify your answer.* $\boxed{\det A=0}$ because 2 rows are identical.

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5. (25) a. (10) Use determinants to show that $A = \begin{pmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{pmatrix}$ has an inverse.

$$\begin{vmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 3\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 3(1-0) = 3 \neq 0.$$

Since $|A| \neq 0$, A^{-1} exists.

b. (15) Find A^{-1} =

$$\left(\begin{array}{ccc|ccc} -2 & 2 & 3 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow{(1)\leftrightarrow(2)} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ -2 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow{2(1)+(2)} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow{(2)\leftrightarrow(3)} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 3 & 1 & 2 & 0 \end{array}\right)$$

$$\xrightarrow{(2)+(1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & 1 & 1 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 3 & 1 & 2 & 0 \end{array}\right) \xrightarrow{\frac{1}{3}(3)} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & 1 & 1 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1/3 & 2/3 & 0 \end{array}\right) \xrightarrow[-4(3)+(2)]{-4(3)+(1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4/3 & -5/3 & 1 \\ 0 & 1 & 0 & -4/3 & -8/3 & 1 \\ 0 & 0 & 1 & 1/3 & 2/3 & 0 \end{array}\right)$$

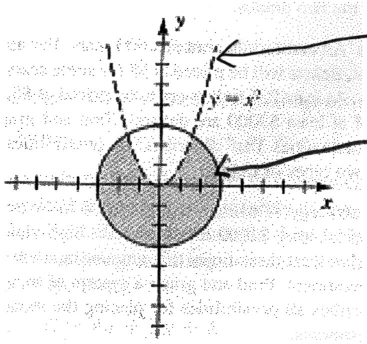
$$\boxed{A^{-1} = \frac{1}{3}\begin{pmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{pmatrix}}$$

6. (10) By expanding the determinants, verify the identity:

$$\begin{vmatrix} a & b \\ kc & kd \end{vmatrix} = k\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

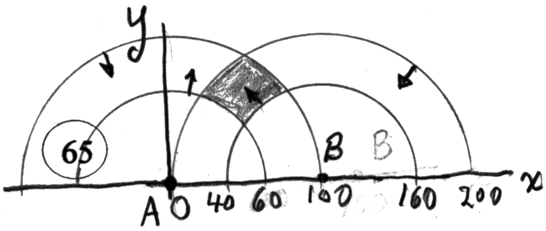
$$\begin{vmatrix} a & b \\ kc & kd \end{vmatrix} = a(kd) - b(kc) = k[ad - bc] = k\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

7. (10) Find a system of inequalities whose graph is shown.



$$\begin{cases} y < x^2 \\ x^2 + y^2 \leq 4^2 \end{cases}$$

8. (20) A nuclear power plant will be constructed to serve the power needs of cities A and B. City B is 100 miles due east of A. The state has promised that the plant will be at least 60 miles from each city. It is not possible, however, to locate the plant south of either city because of rough terrain, and the plant must be within 100 miles of both A and B. Assuming A is at the origin, find and graph a system of inequalities that describes all possible locations for the plant.



$$\begin{cases} x^2 + y^2 \geq 60^2 \\ x^2 + y^2 < 100^2 \\ (x-100)^2 + y^2 \geq 60^2 \\ (x-100)^2 + y^2 < 100^2 \end{cases} \quad y \geq 0$$

9. (20) A linear programming problem: A moose feeding primarily on tree leaves and aquatic plants is capable of digesting no more than 33 kilograms of these foods daily. Although the aquatic plants are lower in energy content, the animal must eat at least 17 kilograms to satisfy its sodium requirement. A kilogram of leaves provides four times as much energy as a kilogram of aquatic plants. Find the combination of foods that maximizes the daily energy intake.

a. (3) What is the objective function?

To maximize $\boxed{E = 4L + A}$, where $\begin{cases} L = \text{no. of kg of leaves eaten} \\ A = \text{no. of kg of a.p. eaten} \end{cases}$

E = daily energy intake

b. (5) Give the constraints.

$$\boxed{\begin{cases} A + L \le 33 \\ A \ge 17 \\ L \ge 0 \end{cases}}$$

c. (12) Solve the linear programming problem.

$A + L = 33 \Rightarrow A = 33 - L$

V_1: (0, 17) – # of L = 0 and A = 17

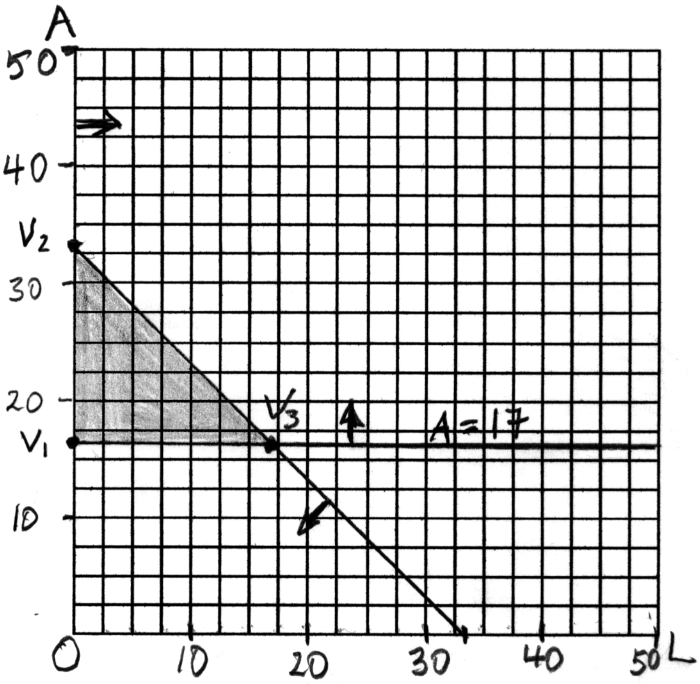
V_2: (0, 33) – A intercept of A + L = 33

V_3: (16, 17) – # of A = 17 and A + L = 33

$\begin{cases} A = 17 \\ A + L = 33 \end{cases} \Rightarrow 17 + L = 33 \Rightarrow L = 16$

Vertex	$E = 4L + A$
V_1: (0, 17)	17
V_2: (0, 33)	33
V_3: (16, 17)	81

E = 81 is maximum at (16, 17)



10. (15) **Set up only the system of equations for solving the following problem. *Do not solve it.***

A shop specializes in preparing blends of gourmet coffees. From Colombian, Brazilian, and Kenyan coffees, the owner wishes to prepare 1-lb bags that will sell for \$8.50. The cost per pound of these coffees is \$10, \$6, and \$8 respectively. The amount of Colombian is to be three times the amount of Brazilian. Find the amount of each type of coffee in the blend.

Let $\begin{cases} c = \text{no. of lb of Columbian coffee} \\ b = \text{" " " " Brazilian "} \\ k = \text{" " " " Kenyan "} \end{cases}$

Total of weights = 1 lb

$\therefore c + b + k = 1$

Total of costs = 8.50

$\therefore 10c + 6b + 8k = 8.50$

Amt of Columbian = 3 (amt of Brazilian)

$c = 3b$

$$\boxed{\begin{cases} c + b + k = 1 \\ 10c + 6b + 8k = 8.50 \\ c = 3b \end{cases}}$$

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