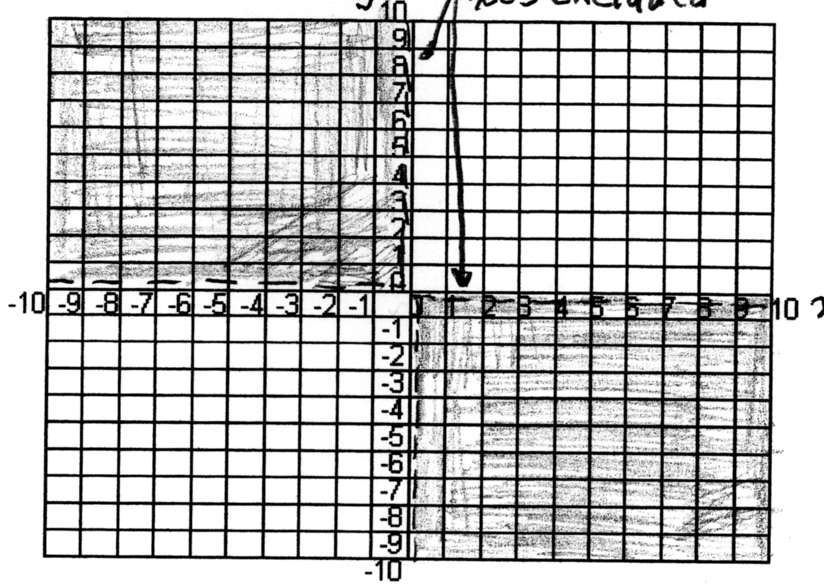
MATH 120A (2410) **Test 2** 2002 Nov 7

Name ANSWER KEY ______points of 145______%

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (5) In the graph below, sketch the set of all points (x,y) for which xy<0.

axes excluded

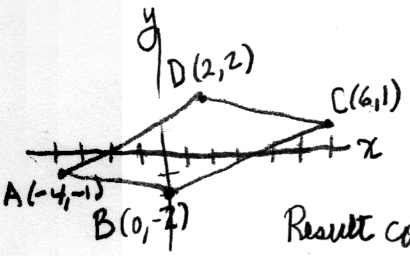


$xy < 0$

$\Leftrightarrow \begin{cases} x > 0 \text{ and } y < 0 \\ \text{or} \\ x < 0 \text{ and } y > 0 \end{cases}$

$\Leftrightarrow (x,y) \in Q\text{II} \cup Q\text{IV}$ excluding axes

2. (8) Show that A(-4,-1), B(0,-2), C(6,1), D(2,2) are vertices of a parallelogram.



Slope $\overline{AD} = \frac{2+1}{2+4} = \frac{3}{6} = \frac{1}{2}$; slope $\overline{BC} = \frac{1+2}{6-0} = \frac{3}{6} = \frac{1}{2}$. $\therefore \overline{AD} \parallel BC$

slope $\overline{AB} = \frac{-1+2}{-4-0} = -\frac{1}{4}$; slope $\overline{DC} = \frac{2-1}{2-6} = -\frac{1}{4}$. $\therefore \overline{AB} \parallel DC$

$\therefore$ quadrilateral ABCD is a parallelogram.

Result could also be shown by proving that $|\overline{AD}| = |\overline{BC}|$ and $|\overline{AB}| = |\overline{DC}|$.

3. (6) Find the center and radius of the circle $x^2+y^2+8x-10y+37 = 0$.

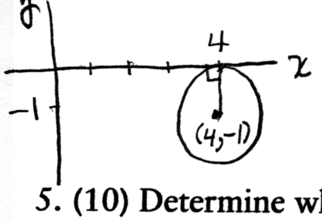
$(x^2+8x) + (y^2-10y) = -37$

$(x^2+8x+16) + (y^2-10y+25) = -37+16+25 = 4$

$(x+4)^2 + (y-5)^2 = 2^2$

$\boxed{\text{center} = (-4,5),\ \text{radius} = 2}$

4. (10) Find the equation of the circle whose center is C(4,-1) that is tangent to the x-axis.



Since the radius through the tangent point is perpendicular to the tangent, as shown in the figure, its length is 1.

$\boxed{(x-4)^2 + (y+1)^2 = 1}$

5. (10) Determine whether or not P(3,8) is within, outside, or on the circle whose center is C(-2,-4) and whose radius is r=13.

Distance between P(3,8) and C(-2,-4) is $\sqrt{(3+2)^2 + (8+4)^2} = \sqrt{25+144}$ $= \sqrt{169} = 13$, which is the length of the radius. $\therefore$ $\boxed{\text{P(3,8) lies on the circle.}}$

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6. (6) For the line 3x+2y = 7, find $2y=-3x+7 \Rightarrow y = \left(-\frac{3}{2}\right)x + \boxed{\frac{7}{2}}$

slope $= \boxed{-3/2}$ slope y-intercept

x-intercept $= \boxed{7/3}$ x-intercept: $3x+2\cdot 0 = 7 \Rightarrow x = 7/3$

y-intercept $= \boxed{7/2}$

7. (5) Find the equation of the line through A(7,-3) that is perpendicular to the line of #6.

Slope of perpendicular line is 2/3, and it passes through (7, -3). ∴ by point-slope form $\boxed{y+3 = 2/3(x-7)} \Leftrightarrow 2x-3y-23=0$

8. (8) Find the equation of the perpendicular bisector of the segment joining A(4,2) and B(-2,10). Perpendicular bisector passes through midpoint of $\overline{AB}$, which is $\left(\frac{4-2}{2}, \frac{2+10}{2}\right) = (1,6)$. It is perpendicular to $\overline{AB}$, which has slope $\frac{10-2}{-2-4} = \frac{8}{-6} = -\frac{4}{3}$.

∴ by point-slope form $\boxed{y-6 = \frac{3}{4}(x-1)} \Leftrightarrow 3x-4y+21=0$.

Eq^{n} could also be found by setting dist of (x,y) to (4,2) = dist of (x,y) to (-2,10).

9. (5) Find the domain of $f(x) = \dfrac{1}{(x-3)\sqrt{x+3}}$.

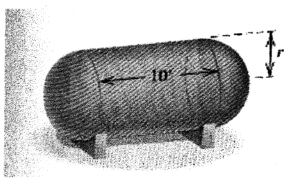
(1) $x-3 \neq 0$, so $x \neq 3$. (2) $x+3 > 0$, so $x > -3$

∴ $\boxed{\text{domain is all reals} > -3, \neq 3}$ or $\boxed{(-3,3) \cup (3,\infty)}$ or $\boxed{\{x \mid x > -3 \text{ and } x \neq 3\}}$

10. (4) Identify as odd, even, or neither, and describe the graphical meaning of the symmetry (if any): f(x) = ½x³+x. $f(-x) = \frac{1}{2}(-x)^3 + (-x) = -\frac{1}{2}x^3 - x = -\left(\frac{1}{2}x^3+x\right) = -f(x)$.

∴ $\boxed{f \text{ is odd}}$ $\boxed{\text{graph of } f \text{ is symmetric w.r.t. origin.}}$

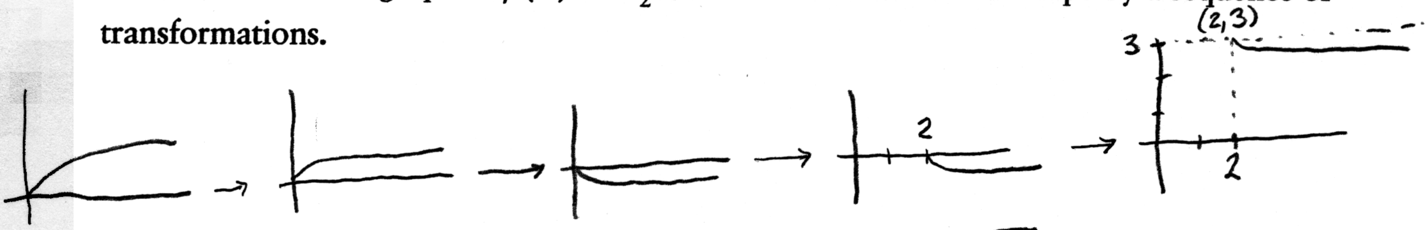
11. (15) A steel storage tank is to be constructed in the shape of a right circular cylinder of altitude 10 feet with a hemisphere attached to each end. The radius r is yet to be determined. Express the volume V (in ft³) of the tank as a function of r (in ft).



Tank consists of cylinder (radius r, h = 10) with two hemispheres (radius r) at each end. Two hemispheres together form a sphere of volume $\frac{4}{3}\pi r^3$; cylinder has volume $\pi r^2 h = 10\pi r^2$.

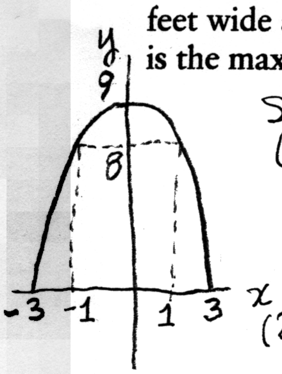
volume of tank = volume of cylinder + volume of sphere. $\boxed{V = 10\pi r^2 + \frac{4}{3}\pi r^3}$

12. (15) Obtain the graph of $f(x) = -\frac{1}{2}\sqrt{x-2} + 3$ from a basic shape by a sequence of transformations.



$y = \sqrt{x}$ Basic shape → $y = \frac{1}{2}\sqrt{x}$ Vertical compression by factor of $\frac{1}{2}$ → $y = -\frac{1}{2}\sqrt{x}$ Reflection about x-axis → $y = -\frac{1}{2}\sqrt{x-2}$ Horizontal translation right by 2 units → $y = -\frac{1}{2}\sqrt{x-2} + 3$ Vertical translation up by 3 units.

13. (8) A doorway has the shape of a parabolic arch and is 9 feet high at the center and 6 feet wide at the base. If a rectangular box 8 feet high must fit through the doorway, what is the maximum width the box can have?



Set up the coordinate system as shown.

(1) To find the eq^n of the parabola. Since 3 and −3 are roots, $y = k(x-3)(x+3)$ $= k(x^2-9)$. $9 = y(0) = k(0^2-9) = -9k \Rightarrow k = -1$. $\therefore y = -x^2+9$.

Eq^n could also be found by noting that because of parabola's position, $y = ax^2+9$. Then $0 = a(\pm 3)^2+9 \Rightarrow a = -1$

(2) Find the intersection of $y=8$ with the parabola. $8 = -x^2+9 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$.

(3) $\therefore$ maximum width of box is $\boxed{2.}$

14. (10) (a.) Show that $f(x) = \dfrac{1}{x+3}$ is one-to-one.

Assume $f(a) = f(b)$

$\Rightarrow \dfrac{1}{a+3} = \dfrac{1}{b+3}$

$\Rightarrow a+3 = b+3$

$\Rightarrow a = b$

$\therefore f$ is one-to-one.

(b.) Find the inverse f^{-1} of f.

Let $y = \frac{1}{x+3} \Rightarrow (x+3)y = 1 \Rightarrow x+3 = \frac{1}{y} \Rightarrow x = \frac{1}{y} - 3$

$\therefore \boxed{f^{-1}(x) = \frac{1}{x} - 3}$

Check: $f \circ f^{-1}(x) = f(\frac{1}{x} - 3) = \frac{1}{(\frac{1}{x}-3)+3} = \frac{1}{1/x} = x$

$f^{-1} \circ f(x) = f^{-1}(\frac{1}{x+3}) = \frac{1}{1/(x+3)} - 3 = (x+3) - 3 = x$

15. (10) Coulomb's law in electrical theory states that the force F of attraction between two oppositely charged particles varies directly as the product of the magnitudes Q_1 and Q_2 of the charges and inversely as the square of the distance d between the particles.

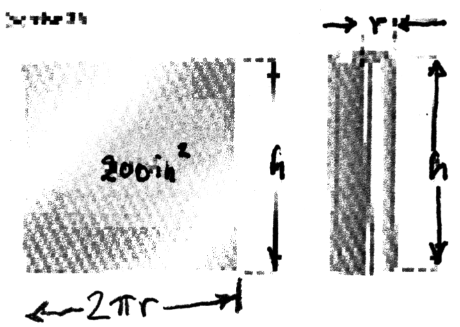
(a.) (4) Find a formula for F in terms of Q_1, Q_2, d, and a constant of variation k.

F varies directly as $Q_1 Q_2$, inversely as d^2. $\therefore \boxed{F = k\dfrac{Q_1 Q_2}{d^2}}$

(b.) (6) What is the effect of reducing the distance d by a factor of ¼ while increasing Q_1 by a factor of 3?

$\begin{cases} d' = \frac{1}{4}d \\ Q_1' = 3Q_1 \end{cases} \Rightarrow F' = k\dfrac{Q_1' Q_2}{(d')^2} = k\dfrac{(3Q_1)Q_2}{(d/4)^2} = 3\left[k\dfrac{Q_1 Q_2}{d^2/16}\right] = 3 \cdot 16\left[k\dfrac{Q_1 Q_2}{d^2}\right] = 48F$

16. (20) Sections of cylindrical tubing are to be made from thin rectangular sheets that have an area of 200 in^2 (see the figure). Is it possible to construct a tube that has a volume of 200 in^3? If so, find r and h.



From the figure, base of rectangle = circumference of circle.

$\therefore$ Area of rectangle = base · height = $2\pi r h$

$\therefore 2\pi r h = 200$

Volume of tube $= \pi r^2 h$ $\therefore \pi r^2 h = 200$

$\begin{cases} 2\pi r h = 200 \\ \pi r^2 h = 200 \end{cases} \Rightarrow \begin{cases} \pi r h = 100 \\ r(\pi r h) = 200 \end{cases} \Rightarrow r(100) = 200 \Rightarrow \boxed{r = 2}$

$\Rightarrow \pi(2)h = 100 \Rightarrow \boxed{h = \dfrac{50}{\pi}}$

Check: Area of rectangle $= 2\pi r h = 2\pi(2)(\frac{50}{\pi}) = 4 \cdot 50 = 200$

volume of tube $= \pi r^2 h = \pi(2^2) \cdot \frac{50}{\pi} = 4 \cdot 50 = 200$

$\boxed{\text{Yes}}$

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