

Name ANSWER KEY \_\_\_\_\_ points of 206 \_\_\_\_\_ %

Write answers and show all work on these sheets. Since partial credit will be given, show sufficient detail. The number of points for each question is shown in parentheses after the number of the question.

1. (20) Simplify the compound fractions:

$$a. \frac{\frac{1}{a+b} - \frac{1}{a}}{b} = \frac{(\frac{1}{a+b} - \frac{1}{a})(a+b)a}{h(a+b)a} = \frac{a - (a+b)}{h(a+b)a} = \frac{-b}{h(a+b)a} = \boxed{-\frac{1}{(a+b)a}}$$

$$b. \frac{3(1+x)^{1/3} - x(1+x)^{-2/3}}{(1+x)^{2/3}} = \frac{[3(1+x)^{1/3} - x(1+x)^{-2/3}](1+x)^{2/3}}{(1+x)^{2/3}(1+x)^{2/3}} \\ = \frac{3(1+x) - x}{(1+x)^{4/3}} = \frac{3+3x-x}{(1+x)^{4/3}} = \boxed{\frac{3+2x}{(1+x)^{4/3}}}$$

$$2. (15) \text{ Solve for } n: S = \frac{n(n+1)}{2}. \Rightarrow 2S = n^2 + n \Rightarrow n^2 + n - 2S = 0 \\ \Rightarrow n = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2S)}}{2 \cdot 1} = \boxed{\frac{-1 \pm \sqrt{1+8S}}{2}}$$

3. (10) Write in the form  $a+bi$ :

$$a. (9-3i) - (-2-6i) = 9-3i+2+6i = \boxed{11+3i}$$

$$b. (9-3i)(-2-6i) = -18 - 54i + 6i + 18i^2 = -18 - 48i - 18 = \boxed{-36-48i}$$

4. (14) Write in the form  $a+bi$ :

$$a. \frac{-2-6i}{9-3i} = \frac{(-2-6i)(9+3i)}{(9-3i)(9+3i)} = \frac{-18-6i-54i-18i^2}{9^2+3^2} = \frac{-18-60i+18}{81+9} = \frac{-60i}{90} \\ = \boxed{-\frac{2}{3}i}$$

$$b. i^{73} = i^{4 \cdot 18 + 1} = (i^4)^{18} i^1 = \boxed{i}$$

$$\begin{array}{r} 18 = Q \\ 4 \overline{) 73} \\ \underline{4} \phantom{0} \\ 33 \\ \underline{32} \\ 1 = R \end{array}$$

5. (40) Solve the equation:

$$\begin{aligned} \text{a. } |2x+5| - 9 &= 11 \\ |2x+5| &= 11+9=20 \\ 2x+5 &= \pm 20 \end{aligned} \quad \left| \begin{aligned} 2x &= -5 \pm 20 \\ x &= \frac{-5 \pm 20}{2} = \boxed{\frac{15}{2}, -\frac{25}{2}} \end{aligned} \right.$$

$$\begin{aligned} \text{b. } x^{4/3} - 5x^{2/3} + 6 &= 0 \\ \text{Let } u &= x^{2/3} \\ u^2 - 5u + 6 &= 0 \\ (u-2)(u-3) &= 0 \end{aligned} \quad \left| \begin{aligned} u &= 2, 3 \\ x^{2/3} &= 2, 3 \\ x &= (x^{2/3})^{3/2} = 2^{3/2}, 3^{3/2} = \boxed{\sqrt{8}, \sqrt{27}} \end{aligned} \right.$$

$$\begin{aligned} \text{c. } x^3 - 5x^2 - 2x + 10 &= 0 \\ x^2(x-5) - 2(x-5) &= 0 \\ (x^2-2)(x-5) &= 0 \\ x^2-2=0 \text{ or } x-5=0 \end{aligned} \quad \left| \begin{aligned} x &= \pm\sqrt{2}, 5 \end{aligned} \right.$$

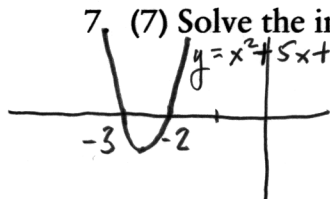
$$\begin{aligned} \text{d. } \sqrt{1+\sqrt{x+\sqrt{2x+1}}} &= \sqrt{5+\sqrt{x}} \\ 1+\sqrt{x+\sqrt{2x+1}} &= 5+\sqrt{x} \quad (\text{square both sides}) \\ \sqrt{x+\sqrt{2x+1}} &= 4+\sqrt{x} \quad (\text{isolate a radical}) \\ x+\sqrt{2x+1} &= 16+8\sqrt{x}+x \quad (\text{square both sides}) \\ \sqrt{2x+1} &= 8(2+\sqrt{x}) \\ 2x+1 &= 64(4+4\sqrt{x}+x) \quad (\text{square both sides}) \\ 2x+1-256-64x &= 256\sqrt{x} \end{aligned}$$

$-[62x+255] = 256\sqrt{x}$   
**No real solution.**  
 Since  $x \geq 0$  for  $\sqrt{x}$  to be real; this makes left side  $< 0$  and right side  $> 0$ .

$$\begin{aligned} \text{6. (5) Solve the inequality: } |2x+5| - 9 &\geq 11. \\ |2x+5| &\geq 11+9=20 \\ 2x+5 &\geq 20 \text{ or } 2x+5 \leq -20 \\ 2x &\geq 15 \text{ or } 2x \leq -25 \end{aligned}$$

$$\begin{aligned} x &\geq 15/2 \text{ or } x \leq -25/2 \\ (-\infty, -25/2] \cup [15/2, \infty) \end{aligned}$$

$$\begin{aligned} \text{7. (7) Solve the inequality: } x^2 + 5x + 6 &> 0. \\ (x+2)(x+3) &> 0 \end{aligned}$$



Parabola opens up,  
 $x$ -intercepts at  $-2, -3$

$$\begin{aligned} x < -3 \text{ or } x > -2 \\ (-\infty, -3) \cup (-2, \infty) \end{aligned}$$

$$\text{8. (15) Solve the inequality: } \frac{x(x-1)}{(x+1)(x-2)^2} \leq 0$$

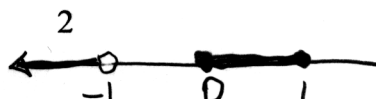
Note: a) can change sign only at  $x = 0, 1, -1$   
 b) Not defined at  $x = 2$  c)  $(x-2)^2$  can be ignored



$x$	$-$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$	$+$
$x+1$	$-$	$+$	$+$	$+$	$+$
$R$	$-$	$+$	$-$	$+$	$+$

$$\begin{aligned} x-1 < 0 &\Leftrightarrow x < 1 \\ x+1 < 0 &\Leftrightarrow x < -1 \end{aligned}$$

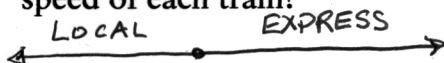
$$\begin{aligned} x < -1 \text{ or } 0 \leq x \leq 1 \\ (-\infty, -1) \cup [0, 1] \end{aligned}$$



In the remaining applied problems:

- List the quantities involved; indicate the values of those that are known, and indicate those which are sought. Alternatively, prepare a table of quantities.
- Make a sketch and label it (if necessary).
- Write in pseudocode any relationships among the quantities.
- Assign a letter to one of the quantities and represent other quantities in terms of this letter (if needed).
- Write an equation needed to solve the problem.
- Solve the equation.
- Check your answer by reference to the original problem; explain in complete sentences why your answer is correct.

9. (20) An express train and a local train start out from the same point at the same time and travel in opposite directions. The express train travels 32 kilometers per hour faster than the local train. If after 3 hours they are 396 kilometers apart, what is the average speed of each train?



Quantities: speed of local (?)  
speed of express (?)  
time of travel (3 hr)  
total distance travelled (396 km)

Distance = rate  $\times$  time

speed of express = speed of local + 32

Total distance travelled = distance travelled by local + dist. travelled by express

Let  $l$  = speed of local

Then  $l + 32$  = speed of express

$$396 = 3l + 3(l + 32)$$

$$396 = 3l + 3l + 96$$

$$6l = 300$$

$$l = 50 = \text{local speed}, l + 32 = 82 = \text{express speed}$$

Check: Local travels  $3 \cdot 50 = 150$  km

Express travels  $3 \cdot 82 = 246$  km

$\therefore$  total distance travelled is  $150 + 246 = 396$ .

10. (20) A merchant blends tea that sells for \$3.00 a pound with tea that sells for \$2.75 a pound to produce 80 lb of a mixture that sells for \$2.90 a pound. How many pounds of each type of tea does the merchant use in the blend?

	Cost/lb	No. of lb	Value
Expensive Tea	3.00	$e$	$3e$
Cheap Tea	2.75	$(80 - e)$	$2.75(80 - e)$
Mixture	2.90	80	$(2.90)(80)$

value of expensive tea + value of cheap tea = total value

$$3e + 2.75(80 - e) = (2.90)(80)$$

$$300e + 22000 - 275e = (290)(80)$$

$$300e + 22000 - 275e = 23200$$

$$25e = 1200$$

$$e = 48$$

48 lb of expensive tea  
32 lb of cheap tea

Check:

Value of expensive tea is

$$48 \times 3.00 = \$144;$$

value of cheap tea is  $32 \times 2.75 = \$88$ .

$\therefore$  value of mixture is  $144 + 88 = \$232$ , which agrees with

$$2.90 \times 80 = 232.$$

(20)

11. A plumber and his assistant work together to replace the pipes in an old house. The plumber charges \$45 per hour for his own labor and \$25 per hour for his assistant's labor. The plumber works twice as long as his assistant on this job, and the labor charge on the final bill is \$4025. How long did the plumber and his assistant work on this job?

Quantities:

Plumber's rate (\$45/hr)

Asst rate (\$25/hr)

Plumber's hours (?)

Asst hours (?)

Plumber's pay

Asst's pay

Pay = Rate  $\times$  Hours      Plumber's hours = 2 (Asst's hours)

Total Pay = Plumber's pay + Asst's pay

Let  $p$  = plumber's hours.

Then  $\frac{1}{2}p$  = asst's hours.

$$4025 = 45p + 25\left(\frac{1}{2}p\right)$$

$$8050 = 90p + 25p$$

$$115p = 8050$$

$$\therefore p = \frac{8050}{115} = 70 = \text{plumber's hours}$$

$$35 = \text{asst's hours}$$

Check: Plumber's pay is  $70 \times \$45 = \$3150$ ;

his asst's pay is  $35 \times \$25 = \$875$ .

$\therefore$  total pay is  $\$3150 + \$875 = \$4025$ , which agrees with final bill.

12. (20) Henry and Irene working together can wash all the windows of their house in 1 h 48 min. Working alone, it takes Henry  $1\frac{1}{2}$  h more than Irene to do the job. How long does it take each person working alone to wash all the windows?

	Time	Rate
Irene	$I$	$\frac{1}{I}$
Henry	$I + \frac{3}{2}$	$\frac{1}{I + \frac{3}{2}}$
Together	$\frac{9}{5}$	$\frac{5}{9}$

Henry's time = Irene's time +  $\frac{3}{2}$

Rate ( $\frac{1}{\text{hr}}$ ) =  $\frac{1}{\text{time}}$

Irene's rate + Henry's rate = combined rate

$$\frac{1}{I} + \frac{1}{I + \frac{3}{2}} = \frac{5}{9}$$

$$\frac{1}{I} + \frac{2}{2I + 3} = \frac{5}{9}$$

$$9(2I + 3) + 2(9)I = 5I(2I + 3)$$

$$18I + 27 + 18I = 10I^2 + 15I$$

$$1 \text{ hr } 48 \text{ min} = 1 + \frac{48}{60} \text{ hr} = 1 + \frac{4}{5} \text{ hr} = \frac{9}{5} \text{ hr.}$$

$$36I + 27 = 10I^2 + 15I$$

$$10I^2 + 15I - 36I - 27 = 0$$

$$10I^2 - 21I - 27 = 0$$

$$I = \frac{21 + \sqrt{(21)^2 - 4(10)(-27)}}{2 \times 10} = \frac{21 + 39}{20}$$

$$= \frac{60}{20} = 3 = \text{Irene's time.}$$

$$4\frac{1}{2} = \text{Henry's time}$$

Check:

Irene's rate is  $\frac{1}{3}$  of the job/hr;

Henry's rate is  $\frac{1}{4\frac{1}{2}} = \frac{1}{9\frac{1}{2}} = \frac{2}{9}$  of the job/hr.

$\therefore$  their combined rate is  $\frac{1}{3} + \frac{2}{9} = \frac{5}{9}$ ,

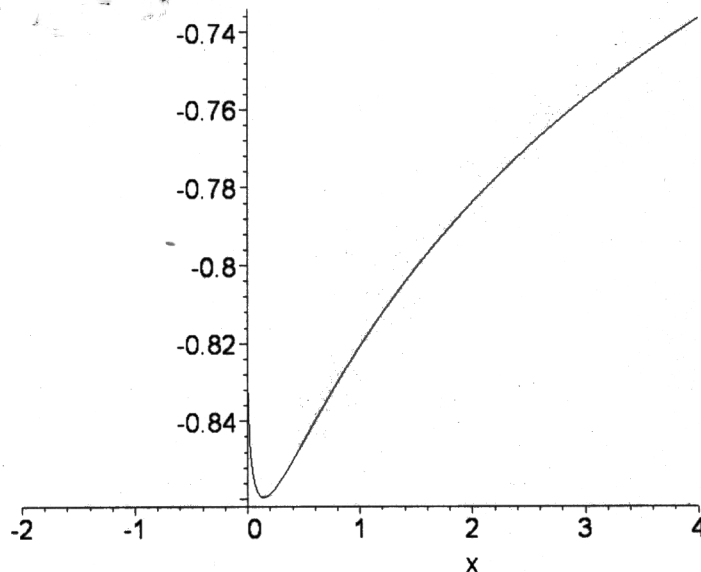
which agrees with the combined rate of  $\frac{5}{9}$ .

#5.d. (p.2)

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> f:=sqrt(1+sqrt(x+sqrt(2*x+1)))-sqrt(5+sqrt(x));  
>
```

$$f := \sqrt{1 + \sqrt{x + \sqrt{2x + 1}}} - \sqrt{5 + \sqrt{x}}$$

```
> plot(f, x=-2..4, color=black);
```



#8, p.2.

```
> g:=x*(x-1)/((x+1)*(x-2)^2);
```

$$g := \frac{x(x-1)}{(x+1)(x-2)^2}$$

```
> plot(g, x=-5..6, y=-5..5, discontin=true, color=black);
```

