

Name: ANSWER KEY

Score: _____

For the polynomial $f(x) = 2x^5 + 5x^4 - 8x^3 - 14x^2 + 6x + 9$

$$f(-x) = -2x^5 + 5x^4 + 8x^3 - 14x^2 - 6x + 9$$

- a. (6) Use Descartes' Rule of Signs to determine the possible number of positive, negative, and complex roots.

pos	neg	∴	ox	11	5	5	5	5
2	3	0		pos	2	2	0	0
1	1	2		neg	3	1	3	1
0	3	2		ox	0	2	2	4
1	1	4						

- b. (4) Find best upper & lower bounds for the roots.

$$\begin{array}{r} 2 \overline{) 2 \quad 5 \quad -8 \quad -14 \quad 6 \quad 9} \\ \underline{4 \quad 18 \quad 20 \quad 12 \quad 36} \\ 2 \quad 9 \quad 10 \quad 6 \quad 18 \quad 45 \end{array}$$

∴ 2 is upper bound

$$\begin{array}{r} -4 \overline{) 2 \quad 5 \quad -8 \quad -14 \quad 6 \quad 9} \\ \underline{-8 \quad 12 \quad -6 \quad 120 \quad -504} \\ 2 \quad -3 \quad 4 \quad -30 \quad 126 \quad -495 \end{array}$$

∴ -4 is lower bound

- c. (6) Find all possible rational roots.

$$c = \pm 9, \pm 1, \pm 3$$

$$d = \pm 2, \pm 1$$

$$\frac{c}{d} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$$

- d. (2) Pare down the list of possible rational roots in part c in accordance with the bounds of part b.

$$\frac{c}{d} = \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$$

- e. (5) Find all possible roots of the polynomial.

$$\begin{array}{r} 1 \overline{) 2 \quad 5 \quad -8 \quad -14 \quad 6 \quad 9} \\ \underline{-2 \quad -7 \quad -1 \quad -15 \quad -9} \\ -1 \overline{) 2 \quad 7 \quad -1 \quad -15 \quad -9 \quad 0} \\ \underline{-2 \quad -5 \quad 6 \quad 9} \\ 2 \overline{) 2 \quad 5 \quad -8 \quad -14 \quad 6 \quad 9} \\ \underline{4 \quad 18 \quad 20 \quad 12 \quad 36} \\ 2 \quad 9 \quad 10 \quad 6 \quad 18 \quad 45 \end{array}$$

∴ 1 is root∴ -1 is root

$$\begin{array}{r} 3/2 \overline{) 2 \quad 5 \quad -8 \quad -14 \quad 6 \quad 9} \\ \underline{3 \quad 15 \quad -12 \quad -21 \quad 24} \\ 2 \quad 8 \quad 6 \quad 7 \quad 9 \end{array}$$

∴ 3/2 is root

$$2x^2 + 8x + 6 = 0$$

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

∴ -3, -1 are roots

- f. (2) Write the polynomial in completely factored form.

$$f(x) = 2(x-1)(x+1)^2(x-\frac{3}{2})(x+3)$$

```
[ > f:=2*x^5+5*x^4-8*x^3-14*x^2+6*x+9;  
                                      $f:=2x^5+5x^4-8x^3-14x^2+6x+9$   
[ > solve(f,x);  
                                     1, -3,  $\frac{3}{2}$ , -1, -1  
[ >
```