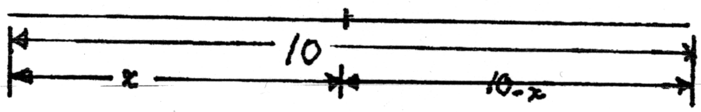
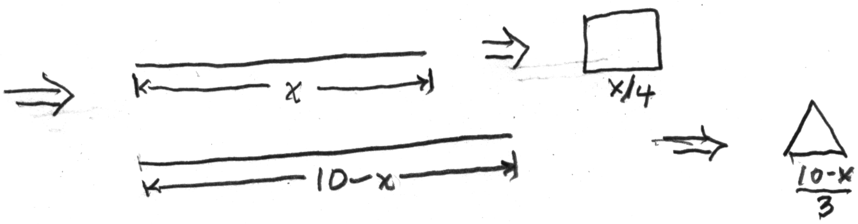
MATH 120A **Quiz 10** 2009 Oct 14

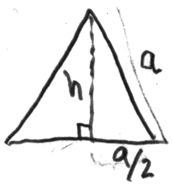
Name: ANSWER KEY Score:________

A piece of wire 10 m long is cut into two pieces. One piece is bent into a square, and the other is bent into an equilateral triangle. Express the sum of the areas of the two figures as a function of the cutpoint x.





Area of □ = Base × height $= \frac{x}{4}\cdot\frac{x}{4} = \left(\frac{x}{4}\right)^2$



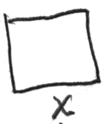
$h^2 + \left(\frac{a}{2}\right)^2 = a^2 \Rightarrow h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \Rightarrow h = \frac{\sqrt{3}}{2}a$

Area of Δ $= \frac{1}{2}$ Base × height $= \frac{1}{2}\cdot\left(\frac{10-x}{3}\right)\cdot\frac{\sqrt{3}}{2}\cdot\left(\frac{10-x}{3}\right)$

$= \frac{\sqrt{3}}{36}(10-x)^2$

∴ Sum of 2 areas $= \boxed{S(x) = \left(\frac{x}{4}\right)^2 + \frac{\sqrt{3}}{36}(10-x)^2}$

Alternative Method:



Area of □ $= x^2$

Area of Δ $= \frac{1}{2}s\cdot\frac{\sqrt{3}}{2}s = \frac{\sqrt{3}}{4}s^2$

∴ Sum of 2 areas $= S = x^2 + \frac{\sqrt{3}}{4}s^2$

Perimeter of □ + perimeter of Δ $= 4x + 3s = 10 \Rightarrow s = \frac{10-4x}{3}$

∴ $\boxed{S(x) = x^2 + \frac{\sqrt{3}}{4}\left(\frac{10-4x}{3}\right)^2}$