

The Fractal Calculus Project

Dr. Mark Meerschaert
Department of Statistics and Probability
Michigan State University

Abstract: Fractional derivatives are almost as old as their integer-order cousins. Recently, fractional derivatives have found new applications in engineering, physics, finance, and hydrology. In physics, fractional derivatives are used to model anomalous diffusion, where a cloud of particles spreads differently than the classical Brownian motion model predicts. A probability model for anomalous diffusion is based on particle jumps with power law tails. The probability of a jump length larger than r falls off like $r^{-\alpha}$ as $r \rightarrow \infty$. For $0 < \alpha < 2$ these particle jumps have infinite variance, indicating a faster than usual spreading rate. Particle traces are random fractals whose dimension α equals the power law tail exponent. A fractional diffusion equation for the concentration of particles $c(x, t)$ at time t and location x takes a form

$$\frac{\partial c(x, t)}{\partial t} = \frac{\partial^\alpha c(x, t)}{\partial x^\alpha}$$

that can be solved via Fourier transforms. Fractional time derivatives model particle sticking or trapping in a porous medium. In finance, price jumps replace particle jumps, and the same models apply. In this talk, we give an introduction to this new area, starting from the beginning and ending with a look at ongoing research. The entire talk will be accessible to advanced undergraduate students.