Saginaw Valley State University

2004 Math Olympics – Level II

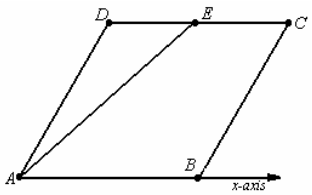
1) If $\log_2(32) - \log_3(27) = \log_4(\frac{x}{5})$, then x is

a) 16 b) 20 c) 80 d) 125 e) None of the above

2) The rhombus *ABCD* is made by gluing two equilateral triangles *ABD* and *BCD* along their common edge, and is placed so that *AB* is along the horizontal *x*-axis. The point E bisects the segment *DC*. Then the slope of the line AE is

a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{2}$ c) 1

d) 1/2 e) None of the above



3) In a local election, 3000 votes were cast to elect one of the three candidates A, B, and C. Of the first 2000 votes, A received 45%, B received 35%, and the rest went to C. Of the remaining 1000 votes, 60% went to C, 25% went to B, and the rest went to A. Which of the following is true?

a) A places first with 60% of the votes b) C wins the election

c) A and C tie for first place d) C received 50 votes more than B

e) None of the above

4) Simplify $\frac{(x^2 - x - 6)(x + 4)}{x^2 + x - 12}$

a) $x + 3$ b) $\frac{x+3}{x+4}$ c) $\frac{x+2}{x+4}$ d) $x + 2$ e) None of the above

5) Let $f(x) = x^2 + 4x + 2$. Let R be the quadrangle (quadrilateral) in the Cartesian plane whose vertices coincide with the *x*-intercepts of *f* and the vertices of the parabolas $y = f(x)$ and $y = -f(x)$. The area of R is

a) $4\sqrt{2}$ b) 12 c) 8 d) $2\sqrt{2}$ e) 2

6) In a class of 60 junior and senior students, 21 are girls, 34 are juniors, and 20 are senior boys. How many juniors are girls?

a) 15 b) 19 c) 6 d) 20 e) 8

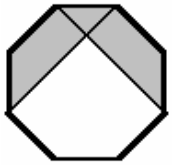
7) A large water tank has two drain outlets A and B. It takes 3 hours to drain the tank through A only, and 4 hours to drain the tank through B only. Suppose that we started draining the full tank using both outlets, but after a while outlet B was closed letting the rest of the water drain through A. Given that it took a total of $2\frac{1}{4}$ hours to drain the tank, how long did it take before closing outlet B?

a) $1\,{}^{1}\!/_{4}$ hrs b) 1 hr c) ${}^{7}\!/_{12}$ hr d) ${}^{5}\!/_{12}$ hr e) None of the above

8) The figure shows a regular octagon of side 1. The area of the shaded region is

a) $\sqrt{2}+\frac{3}{4}$ b) $3\sqrt{2}$ c) $\frac{3\sqrt{2}}{4}$

d) $1+\sqrt{2}$ e) None of the above



9) The length of the side of a regular hexagon with area 1 in^2 is

a) $\sqrt{\frac{3}{4}}$ b) $\frac{3}{4}$ c) $\frac{3\sqrt{2}}{2}$ d) $\sqrt[4]{\frac{3}{4}}$ e) None of the above

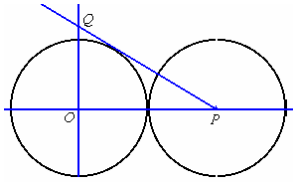
10) If all the roots (zeros) of the polynomial $f(x)=x^5+ax^4+bx^3+cx^2+dx-420$ are integers larger than 1, then $f(4)$ equals

a) 0 b) -6 c) 12 d) -12 e) None of the above

11) In the figure, the two circles both have radius 1 and centers O and P respectively. The line PQ is tangent to the first circle and passes through the center of the second, and the line OQ is perpendicular to the line OP joining the centers. The length of the segment OQ is

a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{2}}{3}$

d) $\frac{2\sqrt{3}}{3}$ e) None of the above



12) If $f(\frac{x}{x-1}) = \frac{1}{x}$ for all $x \neq 0,1$ and $0 < \theta < \pi/2$ then $f(\sec^2\theta) =$

a) $\sin^2\theta$ b) $\cos^2\theta$ c) $\tan^2\theta$ d) $\csc^2\theta$ e) None of the above

13) Suppose that x satisfies the equation $\sin x = \frac{1}{\tan x}$. Then $\cos x$ equals

a) 0 b) $\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{5}-1}{2}$ d) $\frac{\sqrt{5}}{4}$ e) None of the above

14) The domain for the inverse of the function $f(x) = \sqrt{x-5}$ is:

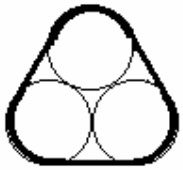
a) $(5, \infty)$ b) The same as the domain of f. c) $[0, \infty)$

d) both a and b are true e) $x = 5$

15) A cashier found that he was often asked to give change for a dollar to people who made no purchase but wanted a dime or two nickels for a parking meter. He started thinking one day about the number of ways he could make change using the coins half-dollar, quarter, dime, nickel, and penny. If he gave no more than four of any coin, in how many different ways could he give change for a dollar to people who needed a dime or two nickels for parking?

a) 6 b) 8 c) 10 d) 12 e) None of the above

16) Three cylindrical drums of 2-foot diameters are to be securely fastened in the form of a triangle by a steel band. What length of band will be required?



a) $6+2\pi$ ft b) 12π ft c) $12 + 2\pi$ ft

d) $6 + 3\pi$ ft e) None of the above

17) A high school runner wants to run a 5-minute mile in his next race. That's an average of 12 miles per hour. He figures he'll start out slowly and finish strong. He runs the first half-mile at 10 miles per hour. How fast must he run the last half-mile so that his time for the race is 5 minutes?

a) 11 mi/hr b) 13 mi/hr c) 14 mi/hr d) 15 mi/hr e) None of the above

18) Using digits from {0,1, … , 9}, a pair of digits (from 00 through 99) is produced as follows. The first digit is randomly selected and registered. A second digit is randomly selected and registered if it differed from the first one. Otherwise a fair coin is flipped. If the coin is heads the digit is registered, and if the coin is tails, a digit is randomly selected and registered. What is the probability that the two registered digits are equal?

a) 0.1 b) 0.055 c) 0.05 d) 0.045 e) None of the above

19) Let $y = mx + b$ be the image when the line $x - 3y + 11 = 0$ is reflected across the *x*-axis. The value of $m + b$ is

a) -6 b) -5 c) -4 d) -3 e) None of the above

20) Let the operation $*$ be defined as $a * b = b + \frac{1}{a}$. The value of $(1 * 2) * 4$ is

a) $3\frac{1}{4}$ b) $2\frac{1}{4}$ c) $4\frac{1}{3}$

d) $1\frac{3}{4}$ e) None of the above

21) One hundred students at Century High School took an exam last year, and their mean score was 100. The number of non-seniors taking the exam was 50% more than the number of seniors, and the mean score of the seniors was 50% higher than that of the non-seniors. What was the mean score of the seniors?

a) 100 b) 112.5 c) 120 d) 125 e) None of the above

22) A survey indicated that in a particular city, 60% of the patients visiting a doctor have a physical disorder, 35% have an emotional disorder, and 25% have both a physical disorder and an emotional disorder. What is the probability of a person having neither a physical nor an emotional disorder visiting a doctor?

a) 0.35 b) 0.30 c) 0.25 d) 0.70 e) None of the above

23) For what values of k will the remainder of division of $x^2 + kx + 4$ by $x - 1$ be twice the remainder of division of $x^2 + kx + 4$ by $x + 1$?

a) 5/3 b) 4 c) ¼ d) 2/3 e) None of the above

24) The solution of the equation $\log(\log x) = 2$ is

a) 10^3 b) 10 c) 10^{10} d) 10^{100} e) None of the above

25) Given the isosceles triangle AEF (where $AE = AF$) with a path of 5 congruent segments A – B – C – D – E – F, the degree measure of angle A is

a) 10° b) 15° c) 20°

d) 30° e) None of the above