2001 MATH OLYMPICS

LEVEL II

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1. The lockers in Pythagoras Middle School are numbered from 1 through 500. Starting the count with locker 1, every sixth locker has a blue decal, every ninth locker has a yellow decal, and every tenth locker has a green decal. How many lockers have all three decals?

   a. 5 b. 6 c. 7 d. 8 e. None of the above

2. Simplify $\dfrac{a - b}{a^{-2} - b^{-2}}$

   a. $a + b$ b. $\dfrac{ab}{a + b}$ c. $\dfrac{1}{a + b}$

   d. $(a^2 - b^2)(a - b)$ e. None of the above

3. The sum of all solutions to the equation $2x^3+5x^2-2x-5=0$ is

   a. -1.5 b. -2.5 c. 2.5 d. -3.5 e. None of the above

4. Three men and two women are waiting to be interviewed for jobs. If they are selected at random, the probability that all the women will be interviewed first is

   a. 40% b. 20% c. 5% d. 10% e. None of the above

5. The sum of the solutions of the equation $\log(x-6)-\log(x+3)=1$ is

   a. 7 b. -7 c. 4 d. -4 e. None of the above

6. Caleb has a balancing scale and finds that a glass and a bowl balance a mug. The bowl alone balances a glass and a plate, and three plates balance two mugs. How many glasses balance a bowl?

   a. 3 b. 4 c. 5 d. 6 e. None of the above

7. The hiking club set out at 12 noon. The members hiked along a level road at a steady rate of 4 mph; up a mountain trail at 3 mph; immediately back down the mountain trail at 6 mph, and back home along the level road, again at 4 mph. They got back at 6:00 p.m. How far did the club hike?

   a. 12 miles b. 18 miles c. 24 miles d. 30 miles e. None of the above

8. If $2^x=y$ then what is $\log_2 \frac{1}{y}$?

   a. x b. $\frac{1}{x}$ c. $\frac{1}{2}$ d. $-x$ e. None of the above

9. As θ increases from 90 degrees to 180 degrees, $\cos\theta$

   a. increases from 0 to 1 b. decreases from 1 to 0 c. increases from -1 to 0

   d. decreases from 0 to -1 e. None of the above

10. The solution set of the inequality $|3x+1| > x+1$ is

    a. $(-\infty,-1/2)\cup(0,\infty)$ b. $(-\infty,0)\cup(1/2,\infty)$ c. $(-\infty,-1/2)\cup(1/2,\infty)$

    d. $(0,1/2)$ e. None of the above

11. If $\sin A \cdot \cos B \geq 1$, then

    a. $\sin A = \cos B$ b. $\sin A = -\cos B$ c. Both (a) and (b) must hold

    d. This can never happen e. None of the above

12. The Pharaoh Chaot IX planned a grand solid pyramid 100m high to be buried under. The builder ran out of stone when it was only 75m high and left it with a flat top. If the builder was paid the prorated price (paid for the part of the job he completed), what fraction of the agreed price did he receive?

    a. 27/64 b. 9/16 c. 3/4 d. 63/64 e. None of the above

13. What is the angle between the hour and minute hands of an ordinary clock showing 7:45 p.m.

    a. 30° b. 37.5° c. 52.5° d. 60° e. None of the above

14. If a committee of 4 is to be chosen from 5 men and 5 women, what is the probability that it will consist of 2 men and 2 women?

    a. 1/2 b. 2/5 c. 10/21 d. 5/63 e. None of the above

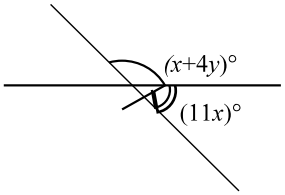
15. A guy wire extends from the top of a 10 foot pole to a point on the level ground 40 feet from the pole. Starting 1 foot from the base of the pole, vertical streamers spaced 1 foot apart extend from the wire to the ground. How many feet of material is required for the streamers?

    a. 210 b. 205 c. 195 d. 190 e. None of the above

16. Given that $x + y = 20$, the value of x in the figure is

    a. 12.5 b. 15 c. 10

    d. 11 e. None of the above



17. If we perform the indicated operations in the expression $\frac{1+\sin t}{\cos t}+\frac{\cos t}{1+\sin t}$ and simplify, then we would obtain

    a. csc$t$ b. tan$t$ c. 2csc$t$ d. 2sec$t$ e. None of the above

18. A circular table is pushed into the corner of a square room so that a point P on the edge of the table is 8 inches from one wall and 9 inches from the other wall as shown. Find the radius of the table in inches.

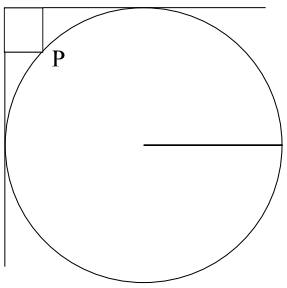
    a. 36 inches

    b. 29 inches

    c. 24 inches

    d. 5 inches

    e. None of the above



19. A cathedral is located at the top of a hill. When the top of the spire is viewed from the base of the hill, the angle of the elevation is 60°. When the spire is viewed from a distance of 200 feet from the base of the hill, the angle of elevation 45°. If the hill rises at an angle of 30°, then the height, in feet, of the cathedral is

a. $200-\frac{100\sqrt{3}}{6}$ b. $100+\frac{200\sqrt{3}}{6}$ c. $100+\frac{100\sqrt{3}}{6}$

d. $200+\frac{200\sqrt{3}}{3}$ e. None of the above

20. Two points are randomly and simultaneously selected from the 4 × 5 grid of 20 lattice points $\{(m,n): 1\le m\le 5, 1\le n\le 4$, with m and n integers$\}$. What is the probability that the distance between them is a rational number?

a. 7/19 b. 36/95 c. 1/2 d. 10/19 e. None of the above

21. The solution, x, of the equation $2^{x+1}=3^{2x-3}$ is

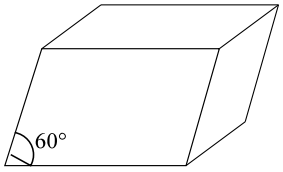
a. $\frac{\log 2+3\log 3}{\log 3-2\log 2}$ b. $\frac{2\log 2+\log 3}{3\log 3-\log 2}$ c. $\frac{2\log 2+\log 3}{\log 3-3\log 2}$

d. $\frac{\log 2+3\log 3}{2\log 3-\log 2}$ e. None of the above

22. The sum of the roots of $x^3+4x^2-7x-10=0$ is -4. What is the sum of the roots of $(x-3)^3+4(x-3)^2-7(x-3)-10=0$?

a. -13 b. -7 c. -1 d. 5 e. None of the above

23. Find the volume of a rhombic prism, that is, a prism with six identical rhombuses as faces, whose faces have edge lengths 1 and whose face angles are 60 degrees and 120 degrees.



a. $\frac{\sqrt{3}}{2}$ b. $\frac{3\sqrt{3}}{4}$ c. $\frac{3\sqrt{2}}{4}$

d. $\frac{3\sqrt{3}}{8}$ e. None of the above

24. The value of $\sin\left(\arccos\left(-\frac{5}{13}\right)\right)$ is

a. $-\frac{5}{13}$ b. $\frac{5}{13}$ c. $-\frac{12}{13}$ d. $\frac{12}{13}$ e. None of the above

25. Two candles of equal length are lit at 8:00 p.m. One candle takes 6 hours to burn out; the other takes three hours. When is one candle exactly twice as long as the other?

a. 8:30 p.m. b. 9:00 p.m. c. 9:30 p.m.

d. 10:00 p.m. e. None of the above