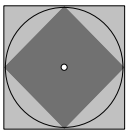
5/9/00 **Math Olympics - Level I Solutions**

These give just one option for solving each problem. Many have numerous methods that will work.

1) (**A**) Slope = $\frac{4-2}{-1-6} = -\frac{2}{7}$, so the equation is $y-2 = -2/7(x-6)$ which simplifies to (A).

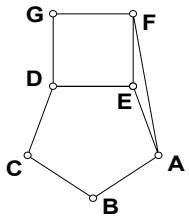
2) (**C**) There are 640/16=40 equal spaces, so 40 + 1 = 41 posts are needed.

3) (**A**) Turning the inside square, as in the picture, we see that the 4 corner triangles make up 2 squares that are ¼ th the area of the larger square. So, subtracting ½ A from A, we have ½ A left.



4) (**D**) The storekeeper lost the $45 change he gave for the radio and the $8 change he gave the man for speaker wire. The storekeeper lost what he/she paid for the two items as well, and the wholesale cost of each was $33 and $7, respectively. This is a total of $93.

5) (**A**) Refer to the picture given. $m\angle FED = 90^\circ$ and $m\angle AED = 108^\circ$ (central angle of the pentagon is 72°), so $m\angle FEA = 360-90-108 = 162^\circ$. Since ΔFEA is isosceles, the base angles are $1/2(180-162) = 9^\circ$.



6) (**E**) Simplifying each side, $6r+3+1 = 27-5r+10 \Rightarrow 6r+4 = -5r+37 \Rightarrow 11r = 33 \Rightarrow r = 3$.

7) (**A**) Multiplying by the reciprocal, $\frac{y^2z}{2y^3}\cdot\frac{8y^4}{9z^5} = \frac{8y^6z}{18y^3z^5} = \frac{4y^3}{9z^5}$.

8) (**B**) The missing number is 47, so the sum of the digits is 11. The pattern is to add to each entry, the entry plus 1.

9) (**B**) $f(x+2) = (x+2)^2+3 = x^2+4x+4+3 = x^2+4x+7.$

10) (**B**) Comparing slopes between pairs of points from a,c,d,e, the slope is always 2. With the point (5,5) the slope is not 2.

11) (**D**) The adjacent angle to the 100° angle is 80°, since they form a straight line. The alternate interior angle to the given 40° angle is also 40°, so the remaining angle in the bottom triangle is $180-80-40 = 60^\circ$. This angle is adjacent to x and they form a straight line, so $x = 120^\circ$.

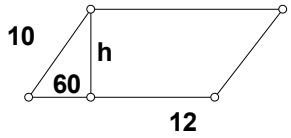
12) (**D**) Since no angle is larger than 85°, the largest angles possible would be 85°. So the smallest possible would be $180-85-85 = 10^\circ$.

13) (**B**) The range of the function $\sqrt{x-4}$ is $[0,\infty)$, so the range of $g(x) = 3+\sqrt{x-4}$ is $[3,\infty)$.

14) (**A**) The probability of drawing a white first and black second is $\frac{6}{10}\cdot\frac{4}{9} = \frac{4}{15}$. The probability of drawing a black first and black second is $\frac{4}{10}\cdot\frac{3}{9} = \frac{2}{15}$. So, the probability of either happening is $\frac{4}{15}+\frac{2}{15} = \frac{2}{5}$.

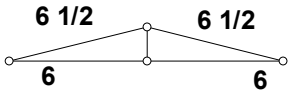
15) **(D)** (i) 40=40 is true. (ii) $3/4 \cdot 72 = 3 \cdot 18 = 54$ is true. (iii) Since 91< 93, $\frac{2}{91} > \frac{2}{93}$ is true. (iv) $3.2 \div 1.6 = 2$, so $3.106 \div 1.6 < 2$. So this one is not true.

16) **(B)** In the 30-60-90 triangle, we have $h = 5\sqrt{3}$, so the area is $12 \cdot 5\sqrt{3} = 60\sqrt{3}$.



17) **(A)** Both numbers are odd, so their sum is even. Therefore, 2 is a factor.

18) **(D)** Using the Pythagorean Theorem, the height is $\sqrt{(13/2)^2 - 6^2} = \sqrt{(13/2 - 6)(13/2 + 6)}$
$= \sqrt{25/4} = 2\,1/2$



19) **(B)** Let d be the distance from the school. Then the walking time (in hours) is $d/4$ and the running time (in hours) is $d/6$. Now, the difference in these times, in hours, is $3\,3/4 \text{ min} \cdot 1/60\, hr/\text{min} = 1/16\, hr$. Solving $d/4 - d/6 = 1/16$, we get $12d - 8d = 3$, or $d = 3/4\, km$.

20) **(C)** At 4:00, Sam has gone 30-4=26 cm and at 5:00, 30-5=25cm more, and at 6:00 has gone 30-6=24 cm more. So, at 6:00 Sam has gone a total of 26+25+24=75cm. Since Sam's speed is 30 cm/hr, it will take him 25/30 hr, or 50 min, to go the remaining 25 cm.

21) **(C)** The figure shown is the minimal number of squares that can be shaded.

22) **(A)** The average of the four nearest neighbors of A and B give the system of equations $A = \frac{64 + 20 + 31 + B}{4}$, $B = \frac{18 + 38 + 54 + A}{4}$. Solving, $4A - B = 115$, $4B - A = 110$, by elimination, $15B = 555$, or $B = 37$.

23) **(D)** From the quadratic formula, we would need the discriminant, $b^2 - 4ac = a^2 - 64 < 0$. Solving this gives (D).

24) **(A)** Setting up the ratio, with x the gallons necessary to go t miles, $\frac{r}{s} = \frac{t}{x}$. Solving gives $x = \frac{st}{r}$.

25) **(C)** $64 = 2^6$, and each man had 64 pennies, so 64 men had a total of $2^6 \cdot 2^6 = 2^{12}$ pennies. Similarly for the women. Therefore, there is a total of $2^{12} + 2^{12} = 2 \cdot 2^{12} = 2^{13}$ pennies.