Saginaw Valley State University

2016 Math Olympics — Level I

1. Suppose $\frac{a}{b}$ is a fraction in the simplest form, where $0 < a < b$, and $b = 2^8 5^5$.
How many decimal places are there to the right of the decimal point in the decimal expansion of $\frac{a}{b}$?

(a) 5 **(b)** 8 **(c)** 13

(d) Not enough information **(e)** None of the above

2. Given that

$$x + y = -5$$
$$x^2 + y^2 = 73$$

find $x^3 + y^3$.

(a) -125 **(b)** -245 **(c)** -365

(d) -485 **(e)** Not enough information

3. How many rational roots does

$$x^{1000} - x^{500} + x^{100} + x + 1 = 0$$

have?

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** None of the above

4. In 2015, one hundred students participated in a math competition, and 20% of them scored at least 20 points. In 2016, one hundred twenty students participated in the competition, and 25% of them scored at least 20 points. What was the percent increase in the number of students who scored at least 20 points in the competition between 2015 and 2016?

(a) 0% **(b)** 5% **(c)** 20% **(d)** 50% **(e)** None of the above

5. Which of the following is a factor of $t^2 - u^2 + 4t + 2u + 3$?

(a) $2t + u$ **(b)** $t + u + 1$

(c) $t + u + 3$ **(d)** The polynomial does not factor

(e) None of the above

6. Which of the following is equal to the sum

$$\frac{2}{3}+\frac{2}{8}+\cdots+\frac{2}{k^2-1}+\cdots+\frac{2}{2016^2-1}$$

(a) $\dfrac{2\left((2016^2-1)!+(2015^2-1)!+\cdots+15!+8!+3\right)}{(2016^2-1)!}$

(b) $\dfrac{7\cdot 2016-10}{2016\cdot 2017}$

(c) $\dfrac{3}{2}-\dfrac{4033}{2016\cdot 2017}$

(d) $\frac{2015}{2016}$

(e) None of the above

7. $\sqrt{\dfrac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}}=$

(a) $\sqrt{2}+1$ **(b)** $\sqrt{2}-1$ **(c)** $\frac{1}{\sqrt{2}}-1$ **(d)** $\frac{1}{\sqrt{2}}+1$ **(e)** None of the above

8. George walks x meters in twenty minutes. If his rate remains constant how many more meters does he walk in the next thirteen minutes?

(a) $\frac{20x}{13}$ meters **(b)** $\frac{x}{20}+13$ meters **(c)** $\frac{20}{x}+13$ meters **(d)** $\frac{x+13}{20}$ meters

(e) $\frac{13x}{20}$ meters

9. Line 1 has intercepts $(2,0)$ and $(0,-3)$. Line 2 goes through the origin and has slope 1. What is the x coordinate of their point of intersection?

(a) 3 **(b)** 6

(c) $-\frac{6}{5}$ **(d)** The lines don't intersect.

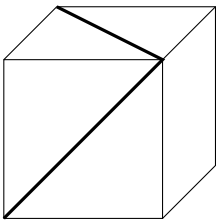
(e) None of the above

10. On which of the following intervals is it true that $|x^2-1|+|x^2-9|$ is always equal to 8?

(a) $-3<x<3$ **(b)** $-3<x<-1$ **(c)** $-1<x<1$ **(d)** $-\infty<x<-3$

(e) $x>1$

11. Shown is a cube and a diagonal on the top face of the cube and a diagonal on the front face of the cube. What is the acute angle between the two diagonals shown?



(a) 45° **(b)** 60° **(c)** 75° **(d)** 90° **(e)** None of the above

12. Which of the following is equivalent to $\sqrt{1\%}$?

(a) 0.1% **(b)** 1% **(c)** 1.1% **(d)** 10% **(e)** 100%

13. What is the ones digit of 2^{2016}?

(a) 0 **(b)** 2 **(c)** 4 **(d)** 6 **(e)** 8

14. What is the acute angle formed by the hour and the minute hands of a correctly working analog clock at 7:24 pm?

(a) 66° **(b)** 72° **(c)** 78° **(d)** 82° **(e)** None of the above

15. When we were preparing this test, we lost the question that went with these answers. Fortunately, since there must be exactly one correct answer for any question, you can decide which one answer below is true.

(a) The cow jumped over the moon.

(b) The cow jumped over the moon, or the little dog laughed to see such a sight.

(c) The little dog laughed to see such a sight, or the dish ran away with the spoon.

(d) The dish ran away with the spoon and the cow jumped over the moon.

(e) If the cow didn't jump over the moon, then the little dog laughed to see such a sight.

16. Suppose $-1 < x < 0$. Which of the following is always true?

(a) $x^3 < x < x^4 < x^2$ **(b)** $x^3 < x < x^2 < x^4$ **(c)** $x < x^3 < x^2 < x^4$

(d) $x^4 < x^3 < x^2 < x$ **(e)** None of the above

17. Two rhinos are charging directly at each other at constant speeds of v and w feet per second, respectively. You foolishly approach them and start your stopwatch when they are d feet apart. Which expression gives the number of seconds before they collide?

(a) $\frac{d}{v}+\frac{d}{w}$ **(b)** $\frac{v+w}{d}$ **(c)** $\frac{d}{v+w}$ **(d)** $\frac{d}{2}v^2+w^2$ **(e)** None of the above

18. Which of the following statements are true for all real numbers x?

I. $\sqrt{x^2}=x$
II. $\sqrt[3]{x^3}=x$
III. $\sqrt{x^4}=x^2$
IV. $\sqrt{x^6}=x^3$

(a) I only **(b)** I and III only **(c)** II only **(d)** II and III only

(e) I, II, III and IV

19. Given a triangle $\triangle ABC$, let G be the centroid of the triangle and l be a line not intersecting any of the sides of the triangle. If the distances of the vertices A, B and C to the line l are 5, 10 and 12 inches, respectively, what is the distance form G to l?

(a) 7 inches **(b)** 9 inches **(c)** 11.5 inches

(d) 13.5 inches **(e)** None of the above

20. Ma Pickle is used to measuring vinegar for her pickle recipe from her blue coffee cup. She uses one and a half blue coffee-cups worth of vinegar. One day she breaks her blue coffee-cup and has to use her red coffee-cup instead. The red coffee-cup is an exact scale model of the blue coffee cup. The blue cup was 5 inches tall and the red cup is 3 inches tall. How many red coffee-cups worth of vinegar will equal one and a half blue coffee-cups of vinegar? (We assume Ma Pickle is good at estimating fractions of cups worth of liquid.)

(a) 9/10 **(b)** 5/2 **(c)** 125/18 **(d)** 250/81 **(e)** None of the above

21. $i^{1887}+\frac{3-i}{1-i}=$

(a) 2 **(b)** 1 **(c)** $2+i$ **(d)** $1-3i$ **(e)** None of the above

22. If $f(x)=ax^2+bx+c$, find b so that the graph of $y=f(x)$ passes through the points $(3,-4)$, $(-1,12)$ and $(2,-3)$.

(a) 6 **(b)** -6 **(c)** 12 **(d)** -12 **(e)** None of the above

23. In a monopoly market, the demand for a product is related to the price by the equation $p = -3x+36$, where x is the number of units demanded and sold each month and p is in hundreds of dollars. Revenue is given by price times the number of units sold. Profit is revenue minus cost. The monthly cost function is given by $C(x) = 2x^2+6x+10$. Find the price that maximizes profit.

(a) \$2700 **(b)** \$300 **(c)** \$3500 **(d)** \$4700 **(e)** None of the above

24. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and homeowners policies will renew at least one of those policies next year.
Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto and homeowners policy.
Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

(a) 20 **(b)** 29 **(c)** 41 **(d)** 53 **(e)** 70

25. Thirty items are arranged in a 6-by-5 array as shown:

$$\begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \\ A_6 & A_7 & A_8 & A_9 & A_{10} \\ A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{16} & A_{17} & A_{18} & A_{19} & A_{20} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{26} & A_{27} & A_{28} & A_{29} & A_{30} \end{array}$$

What is the number of ways to form a set of three distinct items such that no two selected items are in the same row or column?

(a) 200 **(b)** 760 **(c)** 1200 **(d)** 4560 **(e)** 7200