SAGINAW VALLEY STATE UNIVERSITY
2013 MATH OLYMPICS LEVEL I

1. $\sqrt{\frac{1}{9}+\frac{1}{16}}=?$
(a) $\frac{7}{12}$ **(b)** $\frac{1}{5}$ **(c)** $\frac{2}{7}$ **(d)** $\frac{5}{12}$ **(e)** none of the above

2. Suppose the operation $*$ is defined on the set of integers by $a*b=a+2b$. Then for every two integers a and b, the value of $a*(b*a)$ is the same as
(a) $(5a)*b$ **(b)** $a*b$ **(c)** $b*a$ **(d)** $b*(4a)$
(e) none of the above

3. Given that the vertex of the parabola $y=x^2+8x+k$ is on the x-axis, what is the value of k ?
(a) 4 **(b)** 24 **(c)** 16 **(d)** 0 **(e)** 8

4. John and Nancy live on the same street and often walk towards each other's home. If they both leave their homes at 8:00 a.m., then they will meet at 8:04 a.m. If Nancy leaves her home at 8:00 a.m. but John does not leave his home until 8:03 a.m., then they will meet at 8:05 a.m. How many minutes does it take for John to walk all the way to Nancy's home? Assume that each person walks at his or her own constant rate.
(a) 8 minutes **(b)** 9 minutes **(c)** 10 minutes **(d)** 11 minutes
(e) none of the above

5. Which of these numbers is the largest?
(a) $\sqrt{\sqrt[3]{5\cdot 6}}$ **(b)** $\sqrt{6\sqrt[3]{5}}$ **(c)** $\sqrt{5\sqrt[3]{6}}$ **(d)** $\sqrt[3]{5\sqrt{6}}$ **(e)** $\sqrt[3]{6\sqrt{5}}$

6. The graph of the equation $x^2-xy+x-y=0$ is
(a) a parabola **(b)** a point **(c)** an ellipse **(d)** a line
(e) a pair of intersecting lines

7. In a class of 100 students, there are 50 who play soccer, 45 who play basketball, and 50 who play volleyball. Only 15 of these students play all three sports. Everyone plays at least one of these sports. How many of the students play exactly two of these sports?

(a) 20 **(b)** 35 **(c)** 25 **(d)** 15 **(e)** none of the above

8. What is the smallest number of seats in a large auditorium that must be occupied in order to be certain that at least two people share the same first and last initials?

(a) 675 **(b)** 677 **(c)** 51 **(d)** 53 **(e)** none of the above

9. The number of real solutions of the equation $| x - 2 | + | x - 3 | = 1$ is

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** none of the above

10. Suppose that only eight tiles are left in the scrabble bag and the letters on the tiles spell CALCULUS. How many ways (if the order of choosing doesn't matter) can you choose two tiles?

(a) 23 **(b)** 13 **(c)** 20 **(d)** 10 **(e)** none of the above

11. Yannick has a total of \$200 in his two pockets. He takes one fourth of the money in his left pocket and puts it in his right pocket. He then takes \$20 from his left pocket and puts it in his right pocket. If he now has an equal amount of money in each pocket, then how much money did he originally have in his left pocket?

(a) \$120 **(b)**\$160 **(c)** \$180 **(d)** \$140 **(e)** \$80

12. A boat is traveling against the flow of a river. Suppose the river is flowing at a constant speed and the boat maintains a constant speed with respect to the river while traveling in either direction along the river, in other words, the boat would maintain a constant speed in still water. At a certain moment of time a blow-up ball falls off the boat and starts floating down the river. 20 minutes after the ball fell into the water this was noticed and the boat reversed its direction and started going down the river chasing the ball. How long was the ball in the water before it was retrieved?

(a) 20 minutes **(b)** 10 minutes **(c)** 60 minutes
(d) 40 minutes **(e)** none of the above

13. It takes 3 hours filling a pool using two pipes. It takes 5 hours to fill the pool using only the larger pipe. How long does it take to fill the pool using only the smaller pipe?

(a) 1.5 hours **(b)** 4 hours **(c)** 7.5 hours **(d)** 5 hours
(e) none of the above

14. In a certain football league, the only way to score is to kick a field goal for 3 points or score a touchdown for 7 points. Thus the scores such as 1, 4 and 8 are not possible. How many positive scores are not possible?

(a) 5 **(b)** 6 **(c)** 9 **(d)** 11 **(e)** infinitely many

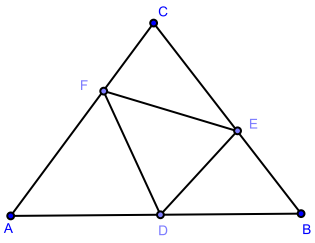
15. For all x such that $x \neq 2, \frac{-2}{5}, 0$, what is $\frac{-24}{4x+8x^2-5x^3} + \frac{1}{2-x}$ equal to?

(a) $\frac{12}{5x^2+2x}$ **(b)** $\frac{5x^2+12x}{2x^2+5x^3}$ **(c)** $\frac{-5}{5x^2+2x}$ **(d)** $\frac{-5-\frac{12}{x}}{5x+2}$
(e) none of the above

16. For how many integers n between 1 and 100 does $x^2 + x - n$ factor into the product of two linear factors with integer coefficients?

(a) 9 **(b)** 2 **(c)** 1 **(d)** 10 **(e)** none of the above

17. On $\triangle ABC$, point D lies on the segment AB, point E lies on BC, and point F lies on CA. If $\frac{AD}{DB} = \frac{BE}{EC} = \frac{CF}{FA} = \frac{2}{3}$, and the area of $\triangle ABC$ equals 1, then what is the area of $\triangle DEF$? Note that the figure below is not drawn to the scale.



(a) $\frac{5}{9}$ **(b)** $\frac{4}{9}$ **(c)** $\frac{3}{25}$ **(d)** $\frac{7}{25}$ **(e)** $\frac{6}{25}$

18. When Alice entered the Forest of Forgetfulness, she forgot the day of the week. She met the Lion and the Unicorn resting under a tree. The Lion lies on Mondays, Tuesdays and Wednesdays and tells the truth on the other days of the week. The Unicorn, on the other hand, lies on Thursdays, Fridays, and Saturdays, but tells the truth on the other days of the week. They made the following statements:

Lion: "Yesterday was one of my lying days."

Unicorn: "Yesterday was one of my lying days."

From these two statements, Alice was able to deduce the day of the week. What day was it?

(a) Monday **(b)** Wednesday **(c)** Thursday **(d)** Friday **(e)** Sunday

19. Suppose that $f(x) = ax^2 - \sqrt{2}$ for some positive real number a. If $f(f(\sqrt{2})) = -\sqrt{2}$, then what is a equal to?

(a) $\frac{\sqrt{2}}{2}$ **(b)** $\frac{2+\sqrt{2}}{2}$ **(c)** $\frac{2-\sqrt{2}}{2}$ **(d)** $2-\sqrt{2}$ **(e)** none of the above

20. From a class of 12 students, 3 are chosen to form a math contest team. The team is required to include at least one boy and at least one girl. If exactly 160 different teams can be formed from the 12 students, then which of the following can be the difference between the number of boys and the number of girls in the class?

(a) 0 **(b)** 2 **(c)** 4 **(d)** 6 **(e)** 8

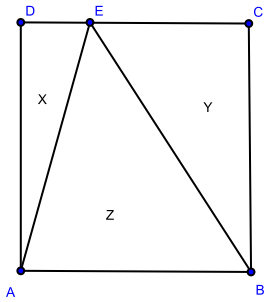
21. An integer valued point in the xy plane is a point (a, b) where both a and b are integers. How many integer valued points are on or inside a circle of radius 4 centered at the origin?

(a) 48 **(b)** 29 **(c)** 60 **(d)** 31 **(e)** none of the above

22. Consider the points $A(-5,-1), B(-1,0), C(1,2)$, and $D(1,3)$. Let P be a point and let $d = PA^2 + PB^2 + PC^2 + PD^2$ so that d is the sum of the squares of the distances from P to each of A, B, C, and D. What is the least possible value for d?

(a) 30 **(b)** 42 **(c)** 36 **(d)** 38 **(e)** 34

23. Suppose that $ABCD$ is a rectangle, and that E is a point on CD. Let X be the area of $\triangle AED$, Y be the area of $\triangle BCE$, and Z be the area of $\triangle ABE$, and suppose that $Y^2 = XZ$. What is the value of $\frac{DE}{EC}$?



(a) $\frac{\sqrt{5}}{3}$ **(b)** $\frac{-1+\sqrt{5}}{2}$ **(c)** $\frac{\sqrt{3}}{2}$ **(d)** $\frac{3}{5}$ **(e)** none of the above

24. The area of a square with diagonal $\sqrt{8}$ is
(a) 8 **(b)** $2\sqrt{2}$ **(c)** 4 **(d)** $\sqrt{2}$ **(e)** none of the above

25. For how many integers m, with $10 \leq m \leq 100$, is $m^2 + m - 90$ divisible by 17 ?
(a) 7 **(b)** 8 **(c)** 9 **(d)** 10 **(e)** none of the above