1. A square is inscribed in a circle, which is inscribed in another square. What is the ratio of the area of the smaller square to the area of the larger square?

    (a) $\frac{1}{2}$ **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $\frac{1}{4}$ **(d)** Impossible to determine from the information given

    (e) None of the above

2. Which of the following choices has a different value from the other choices?

    (a) $32 \cdot 2^{35}$ **(b)** $(32)^8$ **(c)** $\dfrac{2^{40}+2}{2}$ **(d)** $2^{39}+2^{39}$ **(e)** $\dfrac{2^{45}+2^{45}}{2^5+2^5}$

3. At what time between 7:00pm and 8:00pm will the hour hand and the minute hand point in opposite directions? (Assume that the hands move smoothly without jumps.)

    (a) 7 minutes after 7:00pm.

    (b) $6\,{}^5/_{12}$ minutes after 7:00pm.

    (c) $5\,{}^5/_{11}$ minutes after 7:00pm.

    (d) There is no time between 7:00pm and 8:00pm at which the hour hand and the minute hand point in opposite directions.

    (e) None of the above

4. The expression $\dfrac{x^2+2xy-2yz-z^2}{x+2y+z}$ simplifies to

    (a) $2x-2y-z^2$ **(b)** $x-z$ **(c)** $2x-2y-z$

    (d) $x-2y-z$ **(e)** None of the above

5. Which of the following definitions of $x \clubsuit y$ guarantees that $x \clubsuit y = y \clubsuit x$?

    (a) $x \clubsuit y = \dfrac{x}{y}$ **(b)** $x \clubsuit y = 2x+y$ **(c)** $x \clubsuit y = \dfrac{x}{y}+\dfrac{y}{x}$

    (d) $x \clubsuit y = x^2 y$ **(e)** None of the above

6. A kindergarten teacher needs to seat 5 children in a row. The twins, Terry and Sherry, need to sit next to each other or they will cry. How many ways can the teacher seat the 5 children so that Terry and Sherry are next to each other?

   (a) 125   **(b)** 48   **(c)** 24   **(d)** 12   **(e)** None of the above

7. The two smaller circles S_1 and S_2 of radii r_1 and r_2 are initially positioned, respectively, at points P and Q at the opposite ends of a diameter of the big circle of radius r. The smaller circles are then moved along the perimeter of the big circle by rotating them clock-wise (without skidding) at the same rotational rate. After completing 5.5 full rotations, S_1 reaches Q, while at that moment S_2 is only two-thirds of the distance along the arc from Q to P. If the circles continue to rotate until the first moment they are back (simultaneously) to their initial positions, how many rotations would S_2 have made?
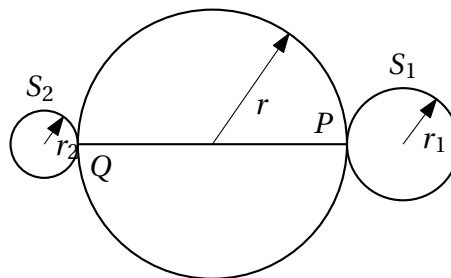
   (a) 27   **(b)** 16.5   **(c)** 70.5   **(d)** 141   **(e)** None of the above

8. $a*b$ is defined to be $a + \dfrac{1}{b}$ for $b \neq 0$. Find $(1*2)*\left(2*\frac{1}{4}\right)$.

   (a) $\frac{5}{3}$ **(b)** $\frac{15}{4}$ **(c)** 2   **(d)** $\frac{10}{9}$ **(e)** None of the above

9. What is the sum of the solutions to the equation $\dfrac{x}{x-1} = 4x^2 + \dfrac{1}{x-1}$?

   (a) 1   **(b)** 1.5   **(c)** 0   **(d)** $\frac{1}{2}$ **(e)** None of the above

10. A lottery game uses a bowl filled with red and white ping-pong balls. Some of the balls have a black dot and some do not. 50% of the balls are colored red, and 20% of the red balls have a black dot. 40% of all the balls have a black dot. What percent of the balls will be white with a black dot?

    (a) 20%   **(b)** 60%   **(c)** 30%   **(d)** 80%   **(e)** None of the above

11. The equation of the tangent line to the circle $x^2 + y^2 = 25$ at $(3, 4)$ is

    (a) $3x + 4y = 25$ **(b)** $3x - 4y = -7$ **(c)** $4x + 3y = 24$

    (d) $4x - 3y = 0$ **(e)** None of the above

12. A worker is pleased to receive a 20% pay rise, but disappointed to have this followed by a 20% pay cut. What is the overall result of this?

    (a) The pay remains the same.   **(b)** 10% cut

    (c) 4% rise   **(d)** 4% cut

    (e) None of the above

13. If a cat and a half can catch a rat and a half in an hour and a half, how many rats can 3 cats catch in 3 hours?

    (a) 9   **(b)** 6   **(c)** 3   **(d)** 2   **(e)** None of the above

14. Donald rents ponies for $15 for one pony. For each additional pony, a $1 per pony discount is given (e.g. 2 ponies rent for $14 each, 3 ponies rent for $13 each etc). What is the most expensive rental Donald can have?

    (a) $225   **(b)** $64   **(c)** $49   **(d)** $144   **(e)** None of the above

15. For $0 < x < 2$, the minimum value of $x + \dfrac{1}{x}$ is

    (a) $\dfrac{3}{4}$ **(b)** 1   **(c)** 2   **(d)** The expression has no minimum value

    (e) None of the above

16. How many gallons of pure alcohol must be added to 10 gallons of a 40% alcohol mixture to get a 50% alcohol mixture?

    (a) 10   **(b)** 5   **(c)** 3   **(d)** 2   **(e)** None of the above

17. A boat can go upstream against a 2.5 mph current in 11 hours, and back downstream to the same place in 7 hours. Find the speed of the boat in still water.

    (a) 5 mph   **(b)** 7 mph   **(c)** 9 mph   **(d)** 13 mph   **(e)** None of the above

18. A tourist travels to Canada and exchanges 1000 American dollars, getting 1250 Canadian dollars. After 4 hours in a casino, he has 100 Canadian dollars left. Assuming the exchange rate did not change during his stay, how much will he get exchanging it for American dollars?

    (a) 75   **(b)** 80   **(c)** 85   **(d)** 90   **(e)** None of the above

19. On Halloween, Tom collects twice as many pieces of candy as Jane, while Mary gets three times as many pieces as Rich. Tom eats half of his candy and gives the other half to Mary. Mary consumes two-thirds what she now has and asks Jane if she wants the rest. Jane opts to take from Mary only as much as Jane originally had. Mary then passes the remaining candy to Rich. Rich ends up with 100 pieces of candy, which is, according to him, 25% more than he originally collected. Which of the following holds?

    (a) The kids collected 550 pieces of candy altogether
    (b) Mary and Jane together collected more than 130% of what Tom and Rich collected together
    (c) Tom and Jane together originally collected 270 pieces
    (d) Mary ended up consuming more candy than she collected
    (e) None of the above

20. What is the equation of the perpendicular bisector of the line segment $(-1, 3)$ to $(3, 5)$? (A line bisects a segment if it divides it into two segments of equal length.)

    (a) $x - 2y = -7$ **(b)** $2x - y = -2$ **(c)** $x + 2y = 13$

    (d) $2x + y = 1$ **(e)** None of the above

21. What is the remainder when $9 \cdot 10^5 + 8 \cdot 10^4 + 7 \cdot 10^3 + 6 \cdot 10^2 + 5 \cdot 10^1 + 4 \cdot 10^0$ is divided by 3?

    (a) 0   **(b)** 1   **(c)** 2   **(d)** 3   **(e)** None of the above

22. Let a be a fixed integer. Alowing repetition, let r_1, r_2 and r_3 be the solutions of the equation $x^3 + ax^2 + ax + 1 = 0$. If r_1 and r_2 are distinct integers, what is the value of r_3?

    (a) $\sqrt[3]{3}$ **(b)** 1   **(c)** $-\frac{1}{2}$ **(d)** $1 + i$ **(e)** None of the above

23. In a strange far-away land, the "star operation" misbehaves: $a \star b \neq b \star a$. Other than that, the operation is associative and $1 \star a = a \star 1 = a$ for any a. Suppose we know that $a \star a \star a = 1$, $b \star b = 1$, and $b \star a = a \star a \star b$. Which of the following must be true about $b \star a \star a$?

    (a) $b \star a \star a = 1$ **(b)** $b \star a \star a = a \star b$ **(c)** $b \star a \star a = a \star a \star b$

    (d) $b \star a \star a = a$ **(e)** None of the above

24. The solution set to the equation $2x^3 - 9x^2 + 25 = 0$ is

    (a) $\left\{-\frac{5}{2}, 1 + \sqrt{6}, 7 - 2\sqrt{6}\right\}$ **(b)** $\left\{\frac{5}{2}, 1 - \sqrt{6}, 7 - 2\sqrt{6}\right\}$ **(c)** $\left\{\frac{5}{2}, 1 - \sqrt{6}, 7 + 2\sqrt{6}\right\}$

    (d) $\left\{-\frac{5}{2}, 1 - \sqrt{6}, 7 + \sqrt{6}\right\}$ **(e)** None of the above

25. Let $n = \sqrt{6 + \sqrt{11}} + \sqrt{6 - \sqrt{11}} - \sqrt{22}$. Then

    (a) $n \geq 1$ **(b)** $0 < n < 1$ **(c)** $n = 0$ **(d)** $-1 < n < 0$

    (e) $n \leq -1$