

# Department of Mathematical Sciences

## Seminar Announcement

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### **The Converse of Lagrange's Theorem**

Thursday, April 7, 2011

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2:45 – 3:45 PM

Science East – Room 124

Lagrange's Theorem states that the size of any subgroup of a finite group is a divisor of the order of the group. Lagrange's Theorem is studied in any introductory abstract algebra class and is undoubtedly one of the most important results in finite group theory. The converse of Lagrange's Theorem is well known to be false. One classical counterexample of smallest order is the alternating group  $A_4$  on four objects. In fact,  $A_4$  is a finite group of order 12 having no subgroup of order 6.

Nevertheless there are groups satisfying the converse of Lagrange Theorem, such groups are called CLT groups. A CLT number is the order of a CLT group and a non-CLT number is number that is not CLT. There are several classes of CLT groups: abelian groups, p-groups, supersolvable groups, and more...

In this talk, we provide different proofs of the facts that  $A_4$  has no subgroup of order 6. We will also show that the group  $SL(2,3)$  (the group of all  $2 \times 2$  invertible matrices with determinant 1 with entries in  $\mathbb{Z}_3$ ) has no subgroup of order 12. The counterexamples  $A_4$  and  $SL(2,3)$  will lead us to consider a related problem to the CLT, namely the characterization of groups with no subgroup of index 2. A solution to the problem would help generate some non-CLT numbers.

I initially designed this talk for my Math 420 (Abstract Algebra) students, but students from Calculus 3 up are welcome to attend. Interested students in doing undergraduate research in that direction under my supervision are encouraged to contact me.