Contemporary Math
Jon Hoerauf
11/21/05

You Can't Get There From Here

This was a fascinating article about the concept of infinity. Apparently, Aristotle had a foundational and long-lasting impact in this area. He not only laid the groundwork for the study of this topic, but his influence also set the limits of investigation for the next 2,000 years. In his view, the concept of something that is infinite (such as the natural numbers) being contained in a set was not "sensible". Therefore, he divided the concept of infinite into tw3o categories; *potential* infinite and *actual* infinite. He would have considered the natural numbers to be *potentially* infinite because you could continue to count and never reach the greatest number. However, he would not consider them to be *actually* infinite because they could not be "contained" within a set.

In 1564 Galileo pondered that if you took an infinite set and removed exactly half of them, the remainder would be as large as the original set. While this happens to be the very definition that today's mathematicians use to define and infinite set, the whole idea was too much for Galileo to continue to consider. By 1675 Isaac Newton and Gottfried Leibnitz began independently developing calculus which involved ideas that required the admission of an actual infinite. However, rather than calling it that, Newton introduced the terminology of, "the ultimate ration of evanescent increments".

In 1874 Georg Cantor completely contradicted the Aristotelian doctrine by proposing actual "completed" infinites. He did this by showing that you could match each number from two sets of infinite numbers (such as whole numbers and even whole numbers). If you matched them one for one you would find out that there is an equal number of members in each set and therefore these sets are the same size.

Cantor next questioned whether or not every infinite set had the same cardinality (or size). He developed a way to check that rational numbers against the real numbers and found that they could indeed be numbered and therefore had the same cardinality. He then checked the real numbers against the rational numbers and found out that, in fact, the set of real numbers was larger.

I read through this article three or four times and wasn't quite able to follow his proof regarding the set of real numbers. In my view, it makes sense to do the proof in the following way. Match the natural numbers and real numbers as follows:

1 – 1.01

2 – 1.001

3 – 1.0001

…

If you match the numbers in this way you would never make it to 2.0. In fact, there would be (dare is say) an infinite number of choices before you even reached 1.2. The whole pattern could even be rearranged in the following manner:

1 – 1.1

2 – 1.11

3 – 1.111

…

Using this format, each of the above options would be an infinite set between 1 and 1.2, and this does not even include 1.12, 1.123, etc… or the "infinite" other options of combinations available. This, to me, proves that the set of real numbers is larger than the set of natural numbers.

Since there are now at least two sets of cardinal numbers, the article placed these cardinals into a set including the finite sets starting with 0. The cardinalities are listed using the Hebrew letter a. The list would be as follows: “cardinals” = {0, 1, 2, 3… a_0, a_1, …?}.

The article referred to a book titled Infinity and the Mind: The Science and Philosophy of the Infinite. I found this topic fascinating and I think that I am going to get this book to read further on it. This article was only an introduction to this topic and I’m sure that the exploration of it could go on, and on, and on, and on, and on…